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SIMULATION OF PARIS-ERDOGAN CRACK PROPAGATION MODEL: THE SIGNIFICANT OF THE POWER VALUE TO STUDY THE EFFECT OF DETERMINISTIC STRESS RANGE ON THE BEHAVIOUR OF DAMAGE OF STRUCTURE

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ABSTRACT

In the field of fatigue failure, the classical Paris-Erdogan model is widely applicable and accepted in fracture mechanics. The model relate da/dN, the crack growth per cycle of applied load, to the parameters of stress-range Δs and the crack length a. The stress range is one of the important parameters in the model which determines the behaviours of damage of any structure. It can be treated as deterministic or random (stochastic) process. In this paper, we study the effect of applying stress sequence in the form of discrete values of ascending and descending order of the same range. The stress used in the model to simulate crack growth is formulated as a function of days and the model is linearised by choosing $M = 2$ as the power value in the model.

1. INTRODUCTION

Fatigue failure due to crack propagation in structures is very important in the field of material engineering. As early as 1930's a large amount of research has been carried out so that reliable models to represent crack propagation could be produced. One of the earliest fatigue crack growth studies was carried out by DeForest [1]. In his paper, DeForest investigated the stress and the number of stress cycles required to start a fatigue crack. Later Langer [2] presented a method for estimating the effects of load cycles of different stresses. He first suggested incorporating models concerning crack propagation into cumulative damage rules. Other works that contributed towards the early development of the cumulative damage approach to fatigue failure include the studies by Thum and Bautz [3], Thum *et al.* [4] and Miner [5].

2. PARIS-ERDOGAN MODEL OF CRACK GROWTH

One of the empirical models which is widely used under conditions of linear-elastic fracture is that proposed by Paris and Erdogan [6]. Although Forman *et al.*, [7] indicated the limitations of Paris's model, we have chosen it as a starting point to develop a simulation model for structural damage or crack growth because of its simplicity. Furthermore, it has been accepted for decades as a basic and widely applicable framework in fracture mechanics [8]. The model relates the increment of crack growth per cycle, da/dn, to the parameters of stress-range Δs and instantaneous crack length a. Paris and Erdogan [6] derived the model based on large range of data and arrived at the expression of the form

$$\frac{da}{dN} = C(\Delta K)^{M} \tag{1}$$

Here $C > 0$, is an empirical crack-growth constant determined by material properties (i.e. elasticity, yield stress and fracture strength). A non-negative material constant, M is a coefficient of model influence, usually cited to lie within the range of $2 \le M \le 4$ [9] and ΔK is the fluctuation range of the crack tip stress-intensity factor K ($\Delta K = K_{\max} - K_{\min}$) which depends upon the size and type of the crack [10]. A simple model for K is given by

$$K = B\sqrt{\pi\, a\, s} \qquad (2)$$

where s is the stress at the crack tip and B is the geometry correction factor which depends on the crack shape, crack length a and the shape of the component. Based on (2) we can have

$$\Delta K = B\sqrt{\pi\, a}\,\Delta s \qquad (3)$$

where Δs is the stress range per load cycle. The value B is taken to be independent of a and often considered as a constant, typically 1.1 [11]. By substituting (3) in (1), the expression has the form

$$\frac{da}{dN} = C(1.1\sqrt{\pi a}\ \Delta s)^M \qquad (4)$$

Taking $\lambda = C(1.1\sqrt{\pi})^M$, equation (4) becomes

$$\frac{da}{dN} = \lambda(\sqrt{a}\ \Delta s)^M \qquad (5)$$

A simulation model to predict crack growth based on equation (5) using a backward difference approximation has been formulated [9]. Hence

$$a_{N+1} - a_N = \lambda\, a_N^{m/2}(\Delta s_N)^M \qquad (6)$$

in which crack growth (damage) accumulates relatively slowly and continuously with the load cycles N [9]. Since M can take any value between 2 and 4, if $M = 2$ is chosen, (6) becomes

$$a_{n+1} = a_n + \lambda\, a_n (\Delta s_n)^2 \qquad (7)$$

If $M = 3$ is chosen, then (6) becomes

$$a_{N+1} = a_N + \lambda a_N^{3/2}(\Delta s_N)^3 \qquad (8)$$

Since most load-bearing structural elements have relatively small initial cracks or fine notches, the initial crack length a_0 must begin with a very small number. Different types of model for the stress range can be used to investigate their effects on crack growth. For each model, an appropriate constant λ need to be chosen so as to get 'readable' results. What is meant by 'readable' is that the results obtained should be meaningful and reasonable, that is, the damages obtained should increase relatively slowly and continuously. The rates of crack growth do not grow too rapidly or abruptly, except after failure occurs. Conventionally, failure is assumed to occur when the damage reaches some critical value. With suitable normalization this value can be taken to be 1.

3. MODELLING THE STRESS RANGE

(3)

The stress range Δs is one of the important parameters in the model which determines the behaviours of damage of any structure. It can be treated as deterministic or random (stochastic) process. The stress sequence used in this model to simulate crack growth is in the form of discrete values of ascending order and descending order of the same range where the stress is formulated as a function of days. By doing this, we can compare the damage and lifetimes of structures or mechanical components experiencing different orders of stress. We are interested in studying the effect of having low or high stress at the initial crack propagation phase to the final damage as well as the lifetime. Since we are going to generate the stress in ascending and descending order of the same range, it can be presented as

(6)

$$s_a = i + c\frac{t}{N} \qquad (9)$$

and

$$s_d = l - c\frac{t}{N} \qquad (10)$$

Equation (9) is for ascending order where the stress starts at initial value i and ends at final value l and equation (10) is of descending order where the stress starts at initial value l and terminates at final value i, $t = 0,1,2,\ldots N$ and N is the total number of days the model is to be run. As a result of the above conditions we have $i + c = l$. In this simulation we decide to take the range from 5 to 10 for s_a and 10 to 5 for s_d. Therefore we have $i = 5$, $l = 10$ and $c = 5$ so that equation (9) becomes $s_a = 5 + 5(\frac{t}{N})$ and equation (10) becomes $s_d = 10 - 5(\frac{t}{N})$.

The crack model is then simulated for values of t from 0 to N and N can take any possible or suitable number of days. Since the parameter λ and initial crack a_0 are arbitrary, suitable values of these constants are chosen in order to generate results that exhibit the characteristic features of interest in crack growth and have to be changed according to the model of equation (7) or (8). For the simulation where M = 2, the parameter values were taken to be $a_0 = 1.0 \times 10^{-1}$ and $\lambda = 0.4 \times 10^{-4}$. For M = 3, they are chosen to be $a_0 = 0.65 \times 10^{-2}$ and $\lambda = 0.5 \times 10^{-4}$. The simulation is carried out for N = 1000. The damages obtained are compared between the two results produced by both stresses, ascending and descending order and for both power values, M = 2 and M = 3. The results obtained are summarized in Table 1 for M = 2 and Table 2 for M = 3 to show the difference at the beginning and at the end of the simulation Only significant days of the simulation are shown in the tables. Each result is then plotted on graphs of damage against number of days respectively in Figure 1 and Figure 2 for M = 2, Figure 3 and Figure 4 for M = 3.

4. RESULTS AND DISCUSSIONS

4.1 For M = 2

Table 1 shows results obtained for the linearised model where M = 2. From the results it is found that, the damage for s_d (as_d) is higher at the earlier part of the structure's life (see Figure 1 and 2). It maintains such behaviour through out but the propagation rate decreases towards the later part of the simulation. As for s_a, the damage propagates very slowly until N is about 800. After this point it increases more rapidly and the propagation rate is at its peak at the end of the simulation. Finally the two crack propagations meet at the end of the simulation. This indicates that the order of the stress is not important for the linearised model. If we examine equation (7), we can see that since the model is linear in a and scalar multiplication is commutative and associative, it follows that the crack size a_n is independent of the ordering of the sequence of stress range. That is the reason why we obtain such result.

Table 1 : Results of damage obtained for ascending order (as_a) and descending order (as_d) of stress

Number of days (N)	dam(asc) (as_a)	dam(desc) (as_d)
0	0.1000	0.1000
1	0.1001	0.1004
2	0.1002	0.1008
971	0.9173	0.9980
972	0.9209	0.9991
973	0.9244	1.0001
992	0.9960	1.0200
993	1.0000	1.0210
994	1.0039	1.0220
999	1.0241	1.0272
1000	1.0282	1.0282

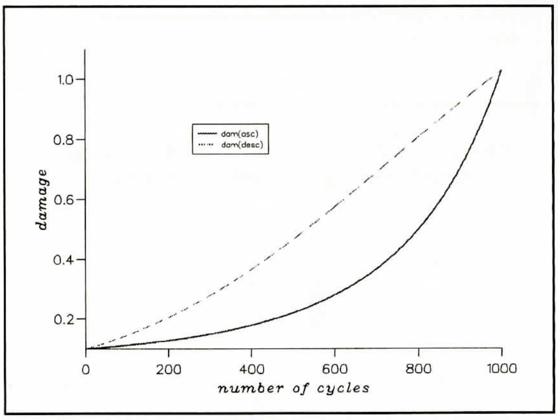

Figure 1 : Graph of damage compared between ascending and descending order of stress

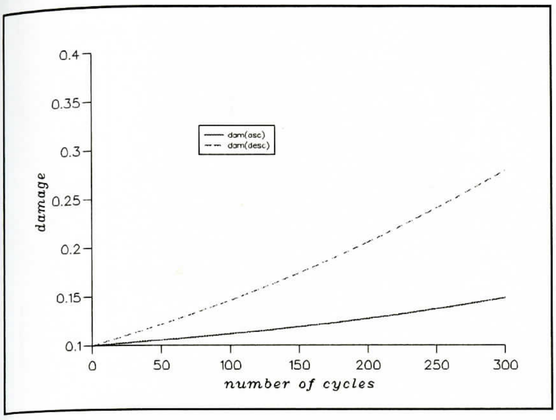

Figure 2 : Graph of damage compared between ascending and descending order of stress at the early part of simulation

4.2 For $M = 3$

From Figure 3 and Table 2 it is found that even though the stress is decreasing for s_d, the damage increases more rapidly because the present damage depends on the previous ones which in turn are raised to the power of 1.5. Therefore s_d gives higher damage and reached 1.0 faster than the damage produced by s_a. (as_a). Figure 4 shows the behaviour of the damage at the early part of the simulation. We can see that the damage simulated from s_d (as_d) propagates more rapidly. Figure 5 shows that when as_d reaches 1.0 at $N = 920$, as_a is still catching up and reached 1.0 at $N = 991$, 70 cycles later. From the results obtained we can conclude that bigger stress or loadings imposed on the structure at the beginning of its life will result in a bigger crack length or damage since initial damage accumulates more rapidly. As a result, the lifetime of the structures is shorter as compared to the one which starts with the lower stress.

It is important to note that, the simulation work is done to study the behaviour of cracks or damage in such condition of stress and the results obtained cannot be compared with the real situation since the data for such behaviour are not available and beyond the reach of the author.

Table 2 : Results of damage obtained for ascending order and descending order of stress

Number of days (N)	dam(asc)	dam(desc)
0	0.006500	0.006500
1	0.006503	0.006526
2	0.006507	0.006553
918	0.141590	0.987260
919	0.143930	0.995030
920	0.146350	1.002900
990	0.973760	1.778600
991	1.021100	1.793800
992	1.072000	1.809300
999	1.566100	1.921600
1000	1.663900	1.938300

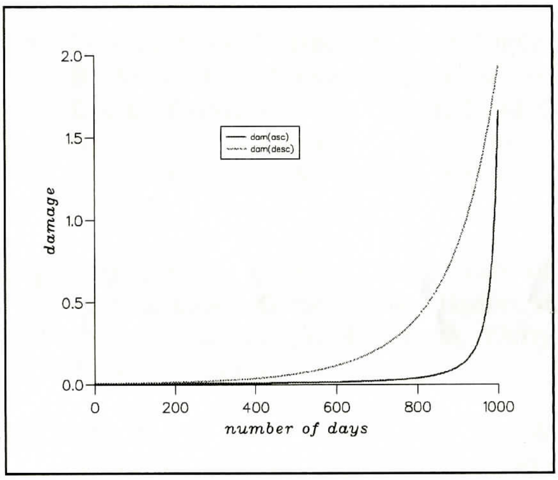

Figure 3 : Graph of damage compared between acsending and descending order of stress

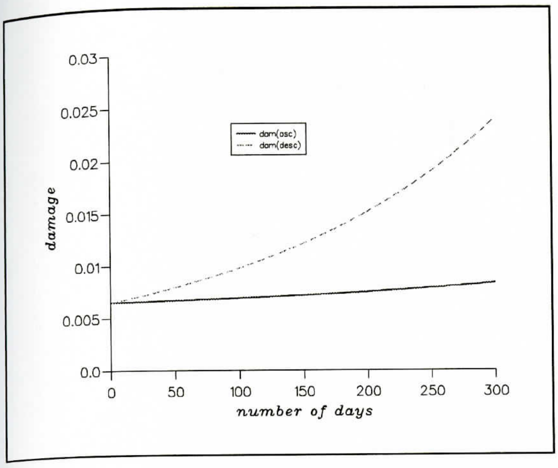

Figure 4 : Graph of damage compared between acsending and descending order of stress at the earlier part of simulation

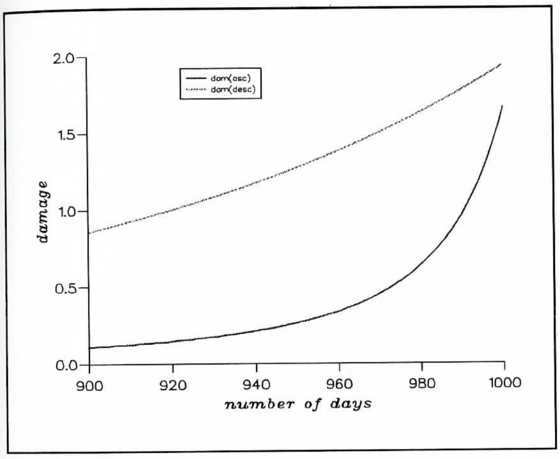

Figure 5 : Graph of damage compared between acsending and descending order of stress at the end of simulation

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