ANTENNA ARRAYS IN MULTI-USER DETECTION OF SPREAD SPECTRUM SIGNALS

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ABSTRACT

In a code division multiple access (CDMA) spread spectrum system, each user is assigned a unique scrambling code. If these codes are not mutually orthogonal, the resulting multiple access interference (MAI) causes frame error rates to be unacceptably high. A number of multi-user detection algorithms have been suggested by various authors. Optimal receivers based upon maximum likelihood sequence decoding are quite complex, their complexity increasing exponentially with the number of users in the system. Sub-optimal receivers such as linear detectors or adaptive cancellers are not as complex, but the error rates increase with the number of users. In this paper, we suggest a new multi-user detection procedure using an antenna array at the receiver. First, we provide an upper bound on the minimum mean-squared error due to MAI, and show how the frame error rate increases with the number of users. The transmitted symbols are estimated by inverting the transfer function of the channel, multiplying it with the outputs of matched filters, and then passing the result through a maximum likelihood decoder. Our analysis indicates that if the channel impulse response is known with sufficient accuracy, this procedure leads to an optimal design. In those cases where the impulse response is only approximately known, we estimate the transmitted symbols and compute the resulting baseband SNR. If this SNR is above a threshold, the baseband signal is acceptable and ready for maximum likelihood sequence decoding. Otherwise, the impulse response is adjusted in small steps until the SNR is above that threshold.

INTRODUCTION

Spread spectrum systems have found wide applications in many areas of military communications. They offer a number of advantages. For example, they are inherently more resistant to jamming. Depending upon the processing gain and the desired error rate, a jamming margin of 10 – 20 dB is not unusual. Another advantage is that without the knowledge of the scrambling code used, an enemy receiver cannot recover the baseband information from the demodulated signal. In addition, spread spectrum signals provide an accurate means for ranging and direction-finding in navigation and radar systems.

If multiple users transmit on the same RF channel, and if the scrambling codes that are used to separate the users in a code division multiple access (CDMA) system are orthogonal, a receiver can correctly decode the baseband signals. However, in most cases, these codes are not purely orthogonal. Signals from various users arrive at a receiver with random propagation delays. In an asynchronous system, these delays may be comparable to the symbol period. As a result, each symbol from any given user may overlap with one or two consecutive symbols from each of the other users. Thus, when the receiver attempts to detect the data from a desired user by multiplying the demodulated signal with a local copy of its scrambling code, contributions to the detected signal from all other users may be quite high, causing significant decoding errors. The resulting interference is called multiple access interference. Multi-user detection is the process of detecting the received signal from a desired user in the presence of this interference.

A number of multi-user detection algorithms have been proposed by various researchers. Some of them are optimal, while others are sub-optimal with higher error rates. The maximum likelihood sequence decoding algorithm proposed by Verdu [1]-[2], [6] is based upon dynamic programming. It uses the outputs of matched filters to estimate a transmitted symbol sequence such that the probability of error over the entire sequence is minimum. Consequently, the algorithm is quite complex, the complexity increasing as 2^K for synchronous systems, where K is the number of users. As for the sub-optimal receivers, there are a few types. For one type, called linear de-correlators, the transmitted symbols are computed by inverting the cross-correlation matrix and then multiplying it with the matched filter outputs [3]- [6]. Clearly, these receivers are not as complex, their complexity being proportional to the number of users. Another type is based upon adaptive cancellation, where the signal from a desired user is decoded by estimating the signal from each

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of the interfering users and successively canceling it from the received signal [5] – [17]. Error rates achievable with these receivers increase with the number of users in the system.

Diversity techniques using multiple antennas have been used in mobile communications to provide diversity [18]. The idea here is that if the amplitude variations of the multipath signals arriving at a receiver are statistically uncorrelated, they can be combined at the receiver so as to maximize the received signal at any instant, thus reducing the possibility of a fade. Clearly, diversity can be used at both transmitters and receivers. Space-time transmit diversity and adaptive antennas have found applications in wideband CDMA (W-CDMA) systems. Multiple-input multiple-output (MIMO) systems, space-time coding (STC) and adaptive antennas are also being utilized in software-defined radios [19] and WiMax technology [20] to improve the system performance.

In this paper, we provide an upper bound on the minimum mean-squared error due to MAI, show how the frame error rate increases with the number of users, and suggest a new multi-user detection procedure using an antenna array at the receiver. The number of antennas is equal to the number of users in the system. The transmitted symbols are estimated by inverting the transfer function of the channel, multiplying it with the outputs of matched filters, and then applying the result to a maximum likelihood decoder. Our analysis indicates that if the channel transfer function is sufficiently accurate, the procedure leads to an optimal receiver. In those cases where the channel is known only approximately or is not known at all, a transfer function is assumed. The performance of the receiver is then optimized by adjusting the transfer function in small steps.

MULTIPLE ACCESS INTERFERENCE

Suppose that K users are transmitting simultaneously on the same radio frequency, each using a separate scrambling code. The transmitted baseband signal from any user k may be represented by

$$x_k(t) = \sum_l s_{k,l}\, c_k(t) p(t - lT) \tag{1}$$

where $s_{k,l}$ is the l-th complex symbol from user k, $c_k(\mathrm{t})$ is the complex scrambling code of that user, $p(t-lT)$ is a raised-cosine filter pulse of duration T starting at instant $t - lT$, and T is the symbol duration. The symbols are chosen from a symbol set $s_{k,l} \in \{\pm 1 \pm j\}$. Each symbol sequence $\{s_{k,l}\}$ is an independent process, each symbol being equally probable with uniform distribution.

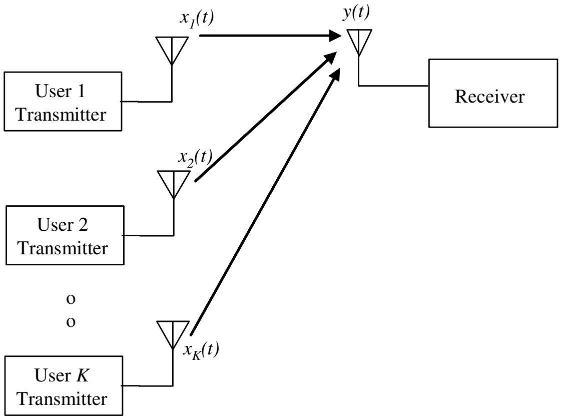

Figure 1 Multiple users in a CDMA system

where each symbol is equally probable with uniform distribution.

In an asynchronous system, the transmitted signals from various users arrive at the receiving antenna over multiple paths with random attenuation and delays. Assuming for simplicity that there is only one path, the received signal may be represented by

$$y(t) = \sum_{k=1}^{K} a_k x_k(t - \tau_k) + \sum_{k=1}^{K} n_k(t) \tag{2}$$

Here a_k is the attenuation and phase delay undergone by the signal from user k, τ_k is the path delay and $n_k(t)$ is the Gaussian noise introduced by the channel. If we want to recover the symbol sequence transmitted by, say, user 1, $y(t)$ is passed through a matched filter, and then multiplied with the complex conjugate of $c_1(t)$, delayed by τ_1. And so, we have

$$r_1(t) = \{s_{1,n}\} + \sum_{k=2}^{K} c_1^*(t - \tau_1) a_k x_k(t - \tau_k) + \sum_{k=1}^{K} c_1^*(t - \tau_1)\, n_k(t) \tag{3}$$

If the scrambling codes are not orthogonal,[1] the middle term on the right-hand side of eq. (3), which represents the multiple access interference to user 1, is non-zero, and with K sufficiently large, because of the central limit theorem, tends to a Gaussian process. Thus, it can be written as

$$r_1(t) = \{s_{1,l}\} + \gamma(t) + n_{1,o}(t) \tag{4}$$

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[1] Two random processes X and Y are orthogonal if E{XY} = 0.

The variance of $\gamma(t)$ depends upon K and the cross-correlation between the scrambling code of user 1 and those of other users.

Eq. (2) can be written as

$$y(t) = a_1(t)x_1(t-\tau_1) + n(t) \tag{5}$$

where $n(t)$ represents the sum total of MAI and channel noise. The transmitted signal $x_1(t)$ from user 1 may be estimated by passing the received signal $y(t)$ through a linear filter and choosing its parameters so as to minimize the mean squared error between the transmitted and estimated signals. In this case, using the orthogonality principle [23], the minimum mean squared error is

$$e_{\min} = S_{X_1X_1}(\omega)\left[1 - \frac{S_{X_1X_1}(-\omega)}{S_{X_1X_1}(\omega) + S_{NN}(\omega)}\right],$$

where $S_{X_1X_1}(\omega)$ and $S_{NN}(\omega)$ are the power spectral density of $x_1(t)$ and $n(t)$ respectively.

If the system is synchronous, all users maintain synchronism using a given time base. Consequently, in this case, only the current symbols from different users overlap with each other. If, on the other hand, the system is asynchronous, each symbol from any user may overlap with two symbols from any other user – the current and the previous one or the current and the next one. As such, the multiple access interference is more severe. This is shown in Figure 2 where we have plotted the frame error rate against SNR for an asynchronous system. Here signals from multiple users have been scrambled with short codes that are only 256 chips long. Notice that if there are only four users in the system, the frame error rate is 0.0015 for an SNR of 6 dB and 0.015 for an SNR of 3 dB. If the SNR falls to 0 dB, the error rate exceeds 0.1 even with only 4 users. If the number of users increases beyond this value, the error rate also begins to increase significantly. In fact, even at an SNR of 6 dB, the frame error rate approaches unity, thus rendering the received data virtually useless.

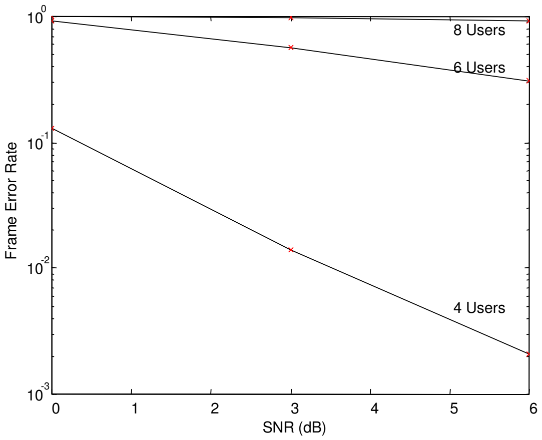

Figure 2 Effect of non-orthogonal scrambling codes

DETECTION USING A RECEIVING ANTENNA ARRAY

Consider the receiver of Figure 3. Let the impulse response of the channel from user k to receive antenna j be denoted by $h_{kj}(t)$ and its discrete-time representation by the sequence

$h_{kj} = \{h_{kj}(0), h_{kj}(1), ..., h_{kj}(M-1)\}$

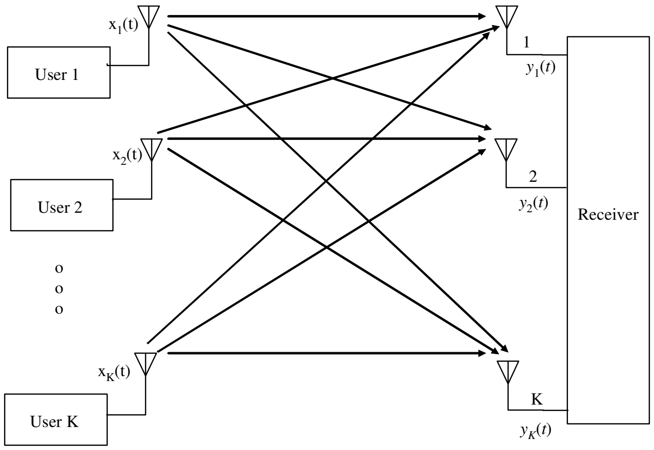

Figure 3 Detection using an antenna array at the receiver

Similarly, assuming that $x_k(i)$ is the value of the transmitted signal from user k at sampling instant i, the signal received at antenna j at instant l, ignoring the noise term, is given by

$$y_j(l) = \sum_{k=1}^{K}\sum_{i=0}^{M-1} h_{kj}(i)x_k(l-i),\ l = 0,1,2,\ldots,L-1. \tag{6}$$

Here L is the length in samples of signals x_k and y_k. Taking the z-transform of both sides, the signal at the receiver is given by

$$\begin{bmatrix} Y_1(z) \\ Y_2(z) \\ . \\ . \\ Y_K(z) \end{bmatrix} = \begin{bmatrix} H_{11}(z) & . & . & H_{K1}(z) \\ H_{12}(z) & . & . & H_{K2}(z) \\ . & . & . & . \quad . \\ H_{1K}(z) & . & . & H_{KK}(z) \end{bmatrix} \begin{bmatrix} X_1(z) \\ X_2(z) \\ . \\ X_K(z) \end{bmatrix} + \begin{bmatrix} n_1(z) \\ n_2(z) \\ \\ . \\ n_K(z) \end{bmatrix} \tag{7}$$

In other words,

$$Y(z) = H(z)X(z) + N_{in}(z) \tag{8}$$

Assuming that the inverse of H exists, we have

$$H^{-1}(z)Y(z) = X(z) + H^{-1}(z)N_{in}(z) \tag{9}$$

Taking the inverse z-transform, we get the discrete transmitted symbols $\{x_k(l)\}$ in the time domain. For each user k, the baseband symbols can now be recovered by simply multiplying x_k with the complex conjugate of its scrambling code $c_k^*(t)$. The symbols thus recovered contain only the Gaussian channel noise, and are decoded using a maximum likelihood decoder. In this case, therefore, the error is minimized over the entire symbol set, thus leading to an optimal receiver. If H is known perfectly, the frame error rate with 8 users in the system is as shown in Figure 4.

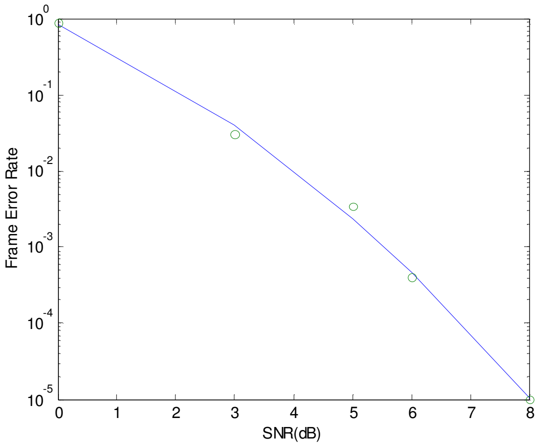

Figure 4 Frame error rate with 8 users

In the above analysis, the impulse response of the channel is assumed to be known. In fact, in many instances, it is possible to characterize the channel with some degree of accuracy. For example, the number of multipaths between a transmitter and receiver depends upon the power delay profile and the associated delay spread. These parameters have been studied for a number of propagation environments [22]. Studies suggest that for many urban areas, two or more multipaths are possible, and a direct path may not be present. In a rural area, there may be a direct path as well as a second, reflected path. Signals arriving at the receiver over different paths undergo different amounts of attenuation and phase delays. Figure 5 shows a channel model, where $x(t)$ is the complex envelope of the transmitted signal, $y(t)$ the complex envelope of the output, $\alpha_i'(t)$ the attenuation and phase of the *i-th* path, $\tau_i(t)$ its delay, ω_c the carrier frequency, and $f_i(t)$ a narrow-band Gaussian process accounting for frequency-selective fading due to the motion of a user device. $f_n(t)$ is obtained by passing white Gaussian noise through a filter whose transfer function is a realizable approximation to

$$G(\omega) = \left[1 - (\omega / \omega_m)^2\right]^{-0.25}.$$

Here ω is the frequency and ω_m the Doppler shift due to the mobile velocity with $|\,\omega\,| \le \omega_m$. The impulse response of the multipath channel is [21] is

$$h(t) = \sum_i \alpha_i'(t) f_i(t) \delta(t - \tau_i(t)).$$

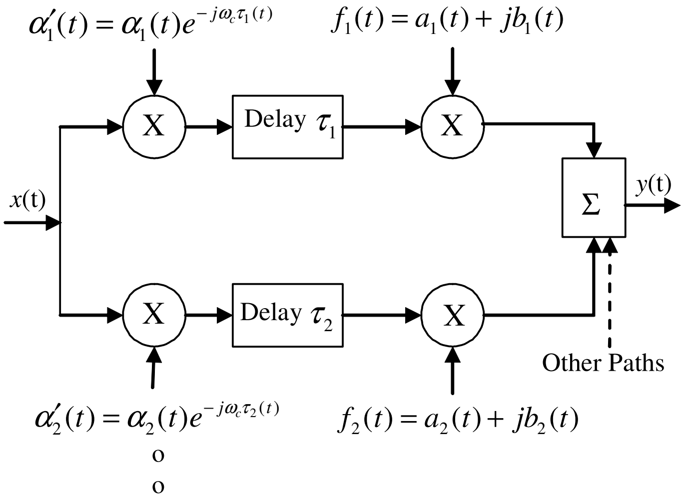

Figure 5 Channel model used

If H is not known accurately, the recovered signal x_k for any user k is not purely de-correlated with respect to signals from other users. To see this, replace H_{ij} by $H_{ij} + \Delta$ in eq. (8), and solve for X. We can then see that the transmitted signal x_k changes by an amount proportional to Δ for all $k \neq i$.[2] If the channel is time-varying, it is necessary to adjust H so as to de-correlate x_k for all k. This is done in the following way.

1. Solve eq. (9) to determine X, multiply x_k for each k by its scrambling code, determine its baseband symbols

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[2] In a sequel to this paper, we will show the effect of an inaccurate and incomplete channel model on the frame error rate, and indicate how we can determine the number of transmit antennas.

and compute the baseband SNR. If this SNR is above a threshold, the baseband signal is acceptable and ready for maximum likelihood sequence decoding.

2. If the SNR is below the threshold, replace H by $H+\Delta H$, determine its baseband symbols and compute the baseband SNR. If the resulting SNR decreases, go to step 3. If the resulting SNR increases and exceeds that threshold, the baseband signal may be applied to the maximum likelihood sequence decoder. If the SNR increases, but is still below the threshold, replace H by $H+\Delta H$, and repeat until the SNR equals or exceeds the threshold.
3. Replace H by $H-\Delta H$, and determine its baseband symbols and compute the baseband SNR. If the resulting SNR increases and exceeds the threshold, the baseband signal may be decoded. If the SNR increases but is still below the threshold, replace H by $H-\Delta H$, and repeat until the SNR reaches or exceeds the threshold.

If a user sends a known data pattern regularly, one can select H so as to minimize the mean squired error e^2 between the transmitted and received signals

$$e^2 = (X - H^{-1}Y)^2 .$$

Taking the partial derivative of this error with respect to element H_{ij}, we have

$$\frac{\partial e^2}{\partial H_{ij}} = -2e\frac{\partial H^{-1}}{\partial H_{ij}}Y$$

Using the gradient algorithm for minimizing e^2, it is necessary to adjust the coefficients of the transfer function as follows [21]:

$$\frac{dH_{ij}}{dt} = -\Delta\frac{\partial e^2}{\partial H_{ij}} = 2\Delta e\frac{\partial H^{-1}}{\partial H_{ij}}Y$$

Here, Δ, the step size, is a positive constant. Thus, the transfer function coefficient H_{ij} at iteration m is given in terms of its value at iteration m -1 by

$$H_{ij}(m) = H_{ij}(m-1) + 2\Delta e\frac{\partial H^{-1}}{\partial H_{ij}}Y^* .$$

In the above equation, Y^* is the complex conjugate of Y. The step size Δ is chosen to be

$$\Delta = \frac{k}{(M+1)Y_{RMS}},$$

where $k \le 1$ is a constant and Y_{RMS} is the RMS value of the observed data sequence.

CONCLUSION

The algorithm we have suggested uses an antenna array at the receiver. The number of receive antennas is equal to the number of users in the system. The transmitted symbols are obtained by multiplying the observed data with an inverse of the channel transfer function and then applying the result to a Maximum likelihood decoder. As such, if the transfer function of the channel is known accurately, the probability of bit error is minimized over an entire symbol set. In that sense, the receiver is optimal. If the function is known only approximately, it is still possible to arrive at an optimal design by adapting the transfer function iteratively using an optimization technique.

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