An automatic time increment selection scheme for simulation of elasto-viscoplastic consolidation of clayey soils

M.R. Karim and F. Oka

During consolidation of geomaterials, pore water flow, deformation, volume change, strain softening or hardening, structural degradation, etc. can occur simultaneously, and consequently leads to time-dependent relations. Therefore, besides spatial discretization, the numerical analysis of the long-term behavior of geomaterials requires temporal discretization which can be solved by any of the numerical methods, such as the finite element method (FEM), even if the material is elastic or rate independent. For rational representation of the behavior of geomaterials, for instance, the elasto-viscoplastic constitutive model (Kimoto et al. 2004), with consideration of the structural degradation effect, can be considered. These advanced formulations can envisage localization phenomena as a result of strong concentration of viscoplastic strain in a distinct narrow zone. Again, inclusion of the soil-structure degradation effect may cause an increase to pore water pressure values even after removing the applied load. This leads to unstable soil behavior (Kabbaj et al. 1988; Oka et al. 1991) where a frequent change in the strain rate is expected. In all cases, the accuracy and stability of the numerical solution are dependent on the choice of time increments as well as on the mesh configuration if the finite element method is used as a numerical tool. However, it seems that only a little attention is paid to finding the proper method for selecting time increments, in contrast to the mesh adaptivity procedures. This is especially true for geomechanical problems involving the elasto-viscoplastic constitutive model.

Usually, the quality of the numerical simulation becomes better if the time domain is discretized through smoother time increment sequences with improved computational stability. To handle a great variety of situations in many complex problems, this can only be achieved through an adaptive or automatic selection approach. Transient computation of viscous flow through porous media provides an application where the adaptivity in time is crucial. The usual temporal error estimates may not be relevant and can be pessimistic when degradation of the structure of the porous medium is dominant. Since the degradation phenomenon is also a transient process, this could result in shorter time increments, thus lessening the computational efficiency. Therefore, the time increment scheme must reflect the numerical stability issues as a prime priority while maintaining reasonable accuracy with optimal computational efficiency. Also, the selection procedure must be able to adapt properly to a wide range of operating conditions including soil conditions.

Time increments can be selected manually if they are relatively small and kept constant. However, a constant smaller time increment over the entire computation time would lead to high computational effort. One can also choose different time increments for different time spans by dividing the entire time domain into various subdomains. However, this selection criterion is strongly dependent on the user’s own experience and prior knowledge about the problem. This is usually difficult to attain, especially in predicting long-term soil response where the time frame of seasonal change and structural degradation of the soil skeleton is not known beforehand. In fact, time increments should at least reflect the soil and model parameters, loading profile, strain rate sensitivity, strain softening or destructuration of the clay structure. As a consequence, proper selection of time increments is important to generate physically correct solutions and at the same time to ensure numerical stability and optimal computational efficiency.

Few adaptive time increment methods are available. Most of them are based on a posteriori temporal error estimation (Diebels et al. 1999; Ehlers et al. 2001). Several alternative approaches are also available such as the method of lines (Matthew et al. 2003), and Proportional-Integral (PI) controlled time-stepping (Soderlind 2002; Valli et al. 1999; Valli et al. 2002). Except for the a posteriori temporal error based methods, most of the other methods have not been applied to multiphase porous media. Recently, Sloan and Abbo (Sloan and Abbo 1999a; Sloan and Abbo 1999b) described an automatic time stepping approach for consolidation problems that subdivides the user specified coarse time increments, by controlling the local error tolerance in the nodal velocity obtained from the difference between the first-order and second-order accurate solutions. A few variants of this approach are also available (Sheng and Sloan 2003). A few empirical guidelines are also available for restricting the maximum time step size depending on the spatial discretization scheme, to minimize the possibility of numerical instability as well as temporal oscillations (Wang et al. 2001; LeVeque 1992).

In the numerical formulation of any time-dependent initial boundary value problem, the time domain can be discretized either by an implicit or explicit scheme or even a combination of both time-marching methods. In addition to the accuracy requirements, time increments are normally restricted by the stability condition of the adopted method. An explicit time integration method is usually more accurate but is stable only when the time increment is chosen sufficiently small, i.e., such methods are conditionally stable (Flanagan and Belytschko 1984). Implicit methods are unconditionally stable. The implicit time integration scheme is well suited to applications where moderate accuracy is needed, and it is the method of preference for most practical analyses. The explicit scheme is more suitable for problems where higher accuracy is required, and can be used to benchmark the accuracy of various integration schemes. Among various explicit schemes, the multistage Runge–Kutta method, especially the classical fourth-order Runge–Kutta scheme, is popular.Various implicit integration schemes are available, such as forward Euler, backward Euler, Crank–Nicolson, including an implicit version of the Runge–Kutta scheme (Brenan et al. 1989).

Because time increments play a crucial role for numerical implementation of any time-dependent boundary value problem, proper selection of time increments is essentially critical for numerical stability and accuracy as well as for computational efficiency. Therefore, a rational procedure for the selection of time increments is necessary. The procedure should possibly be adaptive in nature and capable of incorporating a wide range of material conditions and constitutive relations. It should be unconditionally stable even in simulating an unstable material response. It should also be equally applicable to implicit or explicit time integration schemes, and should not be dependent on the rigid temporal error tolerance criterion. Also, it should be capable of handling arbitrary finite element mesh configurations. In this paper, a robust automatic time increment selection scheme is presented that meets these requirements.

The paper is organized as follows. First, a detailed description of the automatic time increment selection scheme is presented. Later, its performance is investigated by analyzing long-term consolidation behavior of soils under different geotechnical profiles and loading conditions. Both elastic and elasto-viscoplastic constitutive models are considered under one- and two-dimensional plane strain conditions. The finite element method (FEM) is used as a numerical tool and the effectiveness of the proposed scheme is assessed by applying it to both the Euler implicit and Runge–Kutta explicit time integration methods.

The automatic time increment selection scheme

The proposed automatic time increment selection scheme is characterized by the change in time-dependent field variables for the given problem. It is capable of automatically adapting itself to the new conditions depending on the temporal material response. The scheme is dependent on the physical material conditions as well as on the limiting conditions of the constitutive model by which the material is described for numerical implementation. In the present time increment selection scheme, the time increment is assumed to be changing nonlinearly with time from an initial small input value. The time increment for the new time step is determined from the history of the strain rate up to the previous time step. Some control criteria are designed, which are defined by the physical conditions bounded by the constitutive model of the material in consideration. As long as the computed responses are below those bounded or constitutive model-specific failure criteria, time increments continue to grow. However, if any of the control criteria are satisfied, the computation is restarted by going a few time steps back to ensure smoothness of the computed responses, as well as a smooth transition in the change in time increments from large to small values. This procedure repeats itself and ensures temporal stability through continual adjustment of time increments. Even if numerical instability is observed during this process, it is assumed that this instability does not originate from the selected time increment profile. Instead, it is considered that the material has just reached its failure state as set by the working constitutive model, thus the computation has to be stopped unless the governing constitutive model or the mesh configuration is redefined. Note that this failure may not represent the true failure of the material because of the other concerns for numerical instability, including that arising from an improper mesh configuration. At any time step, the automatic time increment selection scheme assumes that the adopted FEM mesh configuration holds good for accuracy and spatial stability.

It is assumed that the strain rate is a good indicator of the current state of geomaterials including their structural degradation. Therefore, it is considered that the time increment profile should reflect the history of the strain rate. Based on this idea, an expression is fitted empirically evolving through the accumulated strain rate invariant by analyzing hundreds of cases of long-term consolidation problems in geomechanics under various soil and loading conditions, elastic and elasto-viscoplastic constitutive models, as well as various manual time increment selection schemes. In the proposed automatic time increment selection procedure, the time increment _t is assumed to change nonlinearly and in a stepwise manner at H user-defined time step intervals (see Figure 1) from an initial user-defined small value _t_c through the expression

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