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Citation: *J. Appl. Phys.* **107**, 043110 (2010); doi: 10.1063/1.3296128

View online: <http://dx.doi.org/10.1063/1.3296128>

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# Superintense fields from multiple ultrashort laser pulses retroreflected in circular geometry

C. H. Raymond Ooi<sup>a)</sup>

School of Engineering, Monash University, Jalan Lagoan Selatan, Bandar Sunway, 46150, Selangor Darul Ehsan, Malaysia

Department of Physics, University of Malaya, 50603 Kuala Lumpur, Malaysia

(Received 30 October 2009; accepted 28 December 2009; published online 26 February 2010)

Laser field with superintensity beyond  $10^{29}$  W/cm<sup>2</sup> can be generated by coherent superposition of multiple 100 fs laser pulses in circular geometry setup upon retroreflection by a ring mirror. We have found the criteria for attaining such intensities using broadband ring mirror within the practical damage threshold and paraxial focusing regime. Simple expressions for the intensity enhancement factor are obtained, providing insight for achieving unlimited laser intensity. Higher intensities can be achieved by using few-cycle laser pulses. © 2010 American Institute of Physics. [doi:10.1063/1.3296128]

## I. INTRODUCTION

Light sources with extreme intensity and power are being actively developed. Chirped pulse amplification (CPA) is the main ingredient for many exciting possibilities and achievements in intense laser science,<sup>1</sup> particularly the synthesis of attosecond<sup>2</sup> laser pulses, laser-plasma interactions,<sup>3</sup> x-ray laser pulses from high-harmonic generation,<sup>4</sup> and the study of electron dynamics in atoms and molecules using ultrafast and ultraintense lasers.<sup>5</sup>

The CPA technique, combined with various amplification schemes, has delivered enormous peak power in petawatt range<sup>6</sup> and intensity around  $10^{22}$  W/cm<sup>2</sup>.<sup>7</sup> In the efforts toward miniaturization of intense laser technology, the technique has incorporated compact tabletop pulsed laser, along with new kinds of solid state lasing materials like ceramics.<sup>8</sup>

Now that the full potential of the CPA technique has been nearly exhausted, new physical processes or techniques are needed to leap into higher intensity range. The quest for nonlinear quantum electrodynamics (QED) science<sup>9</sup> has stimulated efforts on exawatt laser source with intensity beyond  $10^{28}$  W/cm<sup>2</sup> using the mechanism of relativistic compression through interaction with plasma.<sup>10,11</sup>

The maximum intensity  $I (< F_d/T)$  of a laser pulse ( $T$  is pulse duration) from CPA before focusing is limited by the damage threshold  $F_d$  of mirrors or gratings, typically  $0.2-1$  J cm<sup>-2</sup>. One way to increase the intensity is to make shorter pulses but this requires wider bandwidth. Another way is by coherently combining  $N$  (multiple) laser pulses. Coherent interference of multiple lasers gives an enhancement factor of  $N^2$ . This simple concept was briefly mentioned in Refs. 11 and 12, and is used in the National Ignition Facility (NIF) and HIPER facilities for laser fusion. A linear array of equally spaced laser sources is like the diffraction grating, it produces multiple spatial spots with the enhanced field intensity. However, this is inefficient since the peak intensity is not at a single spot. So far, there seems to be no theoretical or experimental study devoted to studying laser

array in circular configuration.

We present a simple scheme/technique that can yield much higher laser intensity at a *single spot* in space by combining the multiple ( $N$ ) noncollinear laser pulses coherently and synchronously. The multiple *laser ports* are situated on a circle (see Fig. 1), each is separated by an angle  $\Theta_o$  such that  $\Theta_n = n\Theta_o (n=1, 2, \dots, N)$ . A ring mirror plays the role of focusing (instead of parabolic mirror), which reflects all the beams toward a single target point C. The lasers are triggered synchronously in such a way that the pulses are released simultaneously toward the center C. Perfect timing is not absolutely critical but would maximize constructive interference at the central region.

## II. LASER PULSES IN CIRCULAR GEOMETRY

The proposed scheme relies on coherent interference of multiple laser pulses and focusing effect from reflection by the ring mirror. We generalize the standard paraxial expression for a Gaussian beam in linear geometry to circular geometry. This involves two-step transformation (rotated and

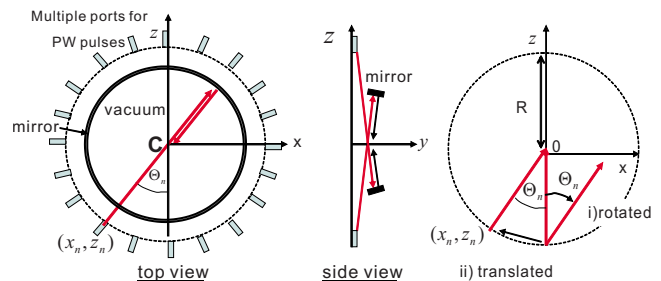


FIG. 1. (Color online) Setup for acquiring superintense laser field. The multiple sources of intense laser pulses are directed synchronously from the circumference of a circle toward the center C (top view). The pulses are directed at a small angle along y-direction (side view) toward a ring mirror. The mirror reflects/focuses the pulses toward C, producing coherently enhanced intensity spot via constructive interference. Transformation coordinates of Gaussian beam to circular geometry is accomplished in two steps: (i) rotation and (ii) translation.

<sup>a)</sup>Electronic mail: bokooi73@yahoo.com.

translated frame of reference) (see Fig. 1). The laser field from the  $n$ -th laser port on the circle of radius  $R$  can be expressed as

$$\mathbf{E}_n(x, y, z, t) = \hat{u}_n E_o e^{-i(\omega\tau_n + \varphi_n)} F(Z_n) \exp[-F(Z_n)\rho_n^2] \times \exp\left[-\left(\frac{\tau_n + \delta_n}{T}\right)^2\right], \quad (1)$$

where  $E_o$  is the field amplitude at the laser ports,

$$F_n = F(Z_n) = i \frac{z_R}{q(Z_n)} = \frac{1}{1 - ia_n} = \frac{1}{\sqrt{1 + a_n^2}} e^{i\Psi(Z_n)},$$

$\rho_n = \sqrt{X_n^2 + y^2}/w_0$ ,  $a_n = Z_n/z_R$ ,  $k_0 = \omega/c$ ,  $\Psi(Z_n) = \tan^{-1} a_n$ , and  $z_R = k_0 w_0^2/2$  is the Rayleigh range. The carrier envelope phase  $\varphi_n$  can be stabilized.<sup>13</sup> The transformed variables are

$$X_n = (x - x_n) \cos \Theta_n - (z - z_n) \sin \Theta_n, \quad (2)$$

$$Z_n = (x - x_n) \sin \Theta_n + (z - z_n) \cos \Theta_n, \quad (3)$$

where  $x_n = -R \sin \Theta_n$ ,  $z_n = -R \cos \Theta_n$  and C is at  $[0,0]$ .

The  $\tau_n = t - Z_n/c$  is retarded time. The effect of (random) errors in the synchronization mechanism enters through  $\delta_n$ . Also,  $T$  is the pulse duration and  $E_o$  is the electric field amplitude at the laser output. The triggering time for release of the laser pulses is at time  $t=0$ . The pulses will meet at point C when  $t=R/c$ . We will plot the intensity distributions across  $x$ - $z$  plane for  $y=0$ . By taking  $y=0$ , we plot the intensity distributions across  $x$ - $z$  plane for linear and circular polarizations. For linear polarization, the unit vector of the field is  $\hat{u}_n = \hat{y}$ . For circular polarization, we have  $\hat{u}_n = (\hat{y} \pm i\hat{e}_n)/\sqrt{2}$ , where  $\hat{e}_n = \hat{x} \cos \Theta_n + \hat{z} \sin \Theta_n$ . These formulae include the beam phases and the angle between the electric field vectors.

The radius  $R$  of the circular laser array is related to the beam waist by

$$R = \alpha N w_0 / \pi, \quad (4)$$

where  $1 < \alpha < 2$  allows for room between two laser ports.

### III. REFLECTION BY RING MIRROR

By using the ABCD law of transformation, we have the electric field after reflection from the mirror (subscript  $m$ )

$$\mathbf{E}_m = \hat{u}_m E_m e^{-i(\omega\tau'_m + \varphi_m)} \mathcal{F}_m \exp\{-\mathcal{F}_m \rho_m'^2\} \times \exp\left[-\left(\frac{\tau'_m + \delta'_m}{T}\right)^2\right], \quad (5)$$

where  $E_m$  is the field amplitude at C after the reflection,

$$\mathcal{F}_m = \mathcal{F}(Z'_m) = i \frac{z_{Rm}}{q_m(Z'_m)} = \frac{1}{\sqrt{1 + b_n^2}} e^{i\Xi(Z'_m)},$$

$b_n = Z'_n/z_{Rm}$ ,  $z_{Rm} = k_0 w_m^2/2$ ,  $q_m(Z'_m) = Z'_m + iz_{Rm}$ ,  $\Xi(Z'_m) = \tan^{-1} b_n$ ,  $\rho_m' = \sqrt{X_n'^2 + y^2}/w_0$ ,  $X_n' = x \cos \Theta_n - z \sin \Theta_n$ ,  $Z_n' = x \sin \Theta_n + z \cos \Theta_n$ , and  $\tau'_m = t - (R + 2R_m)/c + Z'_m/c$ . The width at the center C after reflection is

$$w_m = w_0 \frac{R_m/2z_R}{\sqrt{a^2 + (R_m/2z_R)^2}} = w_0 \sqrt{\alpha}, \quad (6)$$

where  $a = 1 - R_m/2R_1$  and  $R_1 = R + (z_R^2/R)$ ,  $\alpha = z_{Rm}/z_R = (R_m/2z_R)^2/[a^2 + (R_m/2z_R)^2]$ .

The amplitude  $E_m$  has to satisfy the continuity condition  $E_o F_n(R_m) = E_m \mathcal{F}_m(R_m)$  at the reflection points  $(x, y) = R_m(\sin \Theta_n, \cos \Theta_n)$ . After reflection from the mirror, the intensity (at origin C) increases by a factor of

$$\epsilon = \left| \frac{E_m}{E_o} \right|^2 = \left| \frac{f_o}{f_m} \right|^2 = \frac{R_m^2/z_R^2 \alpha^2 + 1}{(R_m + R)^2/z_R^2 + 1}, \quad (7)$$

where

$$f_o = F_n(R_m) = \frac{iz_R}{R_m + R + iz_R}, \quad (8)$$

$$f_m = \mathcal{F}_m(R_m) = \frac{iz_{Rm}}{R_m + iz_{Rm}}. \quad (9)$$

For  $N$  laser pulses arriving at point C simultaneously, the intensity enhancement after reflection from the mirror is

$$\eta = I_m/I_o = N^2 \epsilon \approx N^2 \left( \frac{4z_R}{R_m} \right)^2. \quad (10)$$

Equation (10) shows that the final intensity is boosted by a large number of lasers and a small mirror radius. However, note that  $R_m$  cannot be arbitrarily small due to the reasons discussed below.

### IV. CONSTRAINTS AND OPTIMUM CONDITION

For the typical case of  $z_R = (\pi/\lambda)w_0^2 \gg R$ ,  $R_m$ , we have a simple expression for the enhancement factor  $\epsilon \approx R_m^2/z_R^2 \alpha^2 \approx (4z_R/R_m)^2$  and Eq. (6) simplifies to  $w_m \approx R_m \lambda / 2\pi w_0$  since  $a \approx 1$ . Note that  $R_m$  cannot be arbitrarily small because Eq. (10) is valid within paraxial approximation. Thus, the smallest  $R_m$  is constrained by the paraxial expansion parameter<sup>14</sup>

$$\theta^2 = \left( \frac{\lambda}{\pi w_m} \right)^2 \approx \left( \frac{2w_0}{R_m} \right)^2 \ll 1. \quad (11)$$

In addition, the mirror radius cannot be reduced below a value such that the beams significantly overlap constructively, creating region of intensity beyond the damage threshold. This situation gives  $2\pi R_m = NR_m 2\phi$ , with  $\phi = \sin^{-1}(w_0/R_m)$  as the angle spanned by the laser waist at the mirror. Thus, the minimum radius of the mirror is

$$R_m^{\min} = \frac{w_0}{\sin \phi} = \frac{w_0}{\sin \pi/N}. \quad (12)$$

Let  $2w_0/R_m = 0.1$  [from Eq. (11)] and combine it with Eq. (12), we have

$$R_m^{\min} = w_0 \max \left\{ 20, \frac{1}{\sin \pi/N} \right\}. \quad (13)$$

When  $1/\sin \pi/N < 20$  or  $N < \pi/\sin^{-1}(1/20) = 63$ , the mirror radius is determined by the paraxial condition  $R_m^{\min} = 20w_0$  only depends on the beam waist, and the intensity grows with  $N^2$  according to

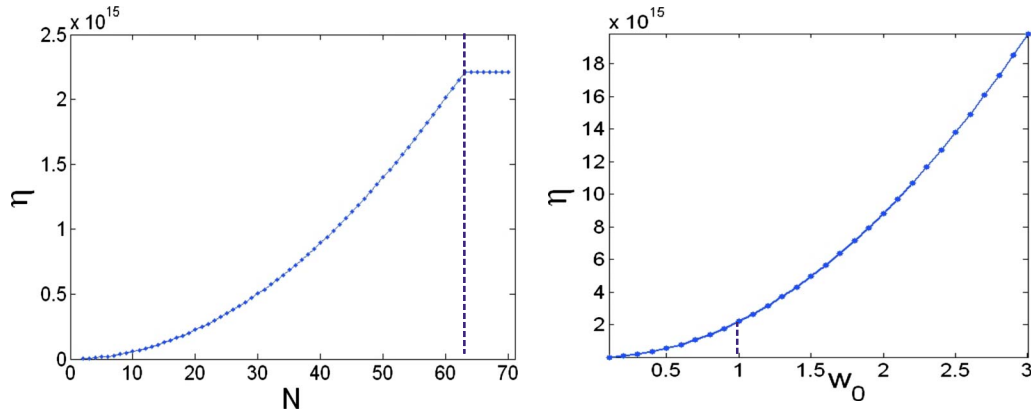


FIG. 2. (Color online) Intensity enhancement factor  $\eta=I/I_o$  plotted using Eqs. (14) and (15). (a)  $\eta$  vs  $N$  with  $w_0=1$  m, and (b)  $\eta$  vs  $w_0$  with  $N=63$ . We have used Eq. (13) for  $R_m$  and  $R=1.1R_m$ .

$$\eta_{<} \approx N^2 \left( \frac{\pi w_0}{5 \lambda} \right)^2. \quad (14)$$

However, when  $N > 63$ , the mirror radius is governed by the circular geometrical condition, which increases almost linearly with  $N$ , i.e.,  $R_m^{\min} = w_0 / \sin \pi/N \approx w_0 N / \pi$ . Here, the corresponding enhancement factor (using Eq. (10)) becomes independent of  $N$ ,

$$\eta_{>} \approx N^2 \left( \frac{4z_R}{w_0 N / \pi} \right)^2 = (2\pi)^4 \left( \frac{w_0}{\lambda} \right)^2. \quad (15)$$

The optimum number of laser beams is 63 since it is not useful to further increase the number of beams. However, Eq. (15) provides an important clue for further intensity enhancement within the paraxial regime, without damaging the mirror. The intensity enhancement is achieved not by increasing the number of laser beams but by using laser beam with large radius, and hence larger circular mirror, since  $R_m \approx w_0 N / \pi$  also increases linearly with  $w_0$ . The mirror will focus the fields to a minimum diffraction limited spot. Thus, the intensity at the spot would be enormous for large beam size which carries more energy.

When  $\eta_{>} = \eta_{<}$ , we obtain an exact analytical expression  $N_o = (2\pi)^2 / (\pi/5) = 20\pi = 62.832$ . The plot of  $\eta$  versus  $N$  (from Eqs. (10) and (13)) in Fig. 2(a) shows the quadratic dependency of the enhancement factor on  $N$  (below  $N_o$ ) and a constant for  $N > N_o$  for  $w_0 = 1$  m. The quadratic increase of the factor with  $w_0$  is shown in Fig. 2(b) for  $N = 63$ .

## V. INTENSITY TOWARD NONLINEAR QED

The laser fluence must be less than the damage threshold of the ring mirror,<sup>15</sup> i.e.,

$$\frac{1}{2} \epsilon_0 E(R_m)^2 c T < F_d \approx 1 \text{ J/cm}^2. \quad (16)$$

which incorporates chirped multilayer dielectric structures<sup>16</sup> to compensate for group velocity dispersion.<sup>17</sup> Similar threshold has been achieved in metal-dielectric grating for pulse compression.<sup>18</sup>

The field amplitude of the beam at the mirror is slightly reduced to  $E_o / \sqrt{1 + (R_m + R)^2 / z_R^2} [= E(R_m)]$  due to diffraction, while the effective waist of a beam at the mirror is slightly increased to  $w_{\text{eff}} = w_0 \sqrt{1 + (R_m + R)^2 / z_R^2}$ . For typical values of

$w_0 \geq 5$  cm and wavelength  $\lambda = 840$  nm, the Rayleigh range is  $z_R \approx 10^5$  m, much larger than typical  $R$  and  $R_m$ . Thus,  $w_{\text{eff}} \approx w_0$  and the peak electric field should be limited by the maximum  $F_d$  and the minimum  $T$ , i.e.,  $E(R_m) \approx E_o < \sqrt{2F_d / \epsilon_0 c T} \approx 8.7 \times 10^9$  V/m for pulse duration  $T = 100$  fs. The corresponding maximum intensity is  $I_o^{\max} = F_d / T = 10^{17}$  W m<sup>-2</sup>, which is below a practical value of  $10^{14}$  W cm<sup>-2</sup>.<sup>19</sup> The energy, power, and intensity of a single beam can be computed from  $U_o = F_d A_o = \frac{1}{2} \epsilon_0 E_o^2 c T A_o$ ,  $P_o = U_o / T$ , and  $I_o = P_o / A_o = F_d / T$  ( $A_o = \pi w_0^2$ ), respectively.

The maximum total energy that can be reflected and focused by the circular mirror is  $U = N U_o$ , where  $N = 63$  is the optimum number of laser beams. The estimated maximum intensity is the ratio of the total energy to the product of effective diffraction limited area  $A_{\text{eff}}$  and pulse duration  $T$

$$I^{\max} = \frac{U}{T A_{\text{eff}}} = \frac{N U_o}{T A_{\text{eff}}} = I_o^{\max} (2\pi)^4 \left( \frac{w_0}{\lambda} \right)^2. \quad (17)$$

Hence, we have the effective (diffraction limited) area of the superintense spot

$$A_{\text{eff}} = \frac{N A_o}{(2\pi)^4 \left( \frac{w_0}{\lambda} \right)^2} = \frac{63 \lambda^2}{16 \pi^3} = 0.127 \lambda^2. \quad (18)$$

For  $N = 63$ ,  $w_0 = 1$  m,  $I_o^{\max} = 10^{17}$  W m<sup>-2</sup>, and  $\lambda = 840$  nm, Eq. (15) correctly gives  $\eta_{>} \approx 2.2 \times 10^{15}$  (as in Fig. 2), with the peak intensity of  $2.2 \times 10^{32}$  W m<sup>-2</sup> sufficient to make the quantum vacuum nonlinear. The estimated result is in agreement with the peak field distribution from full-scale simulation in Fig. 3.

If shorter pulses (smaller  $T$ ) were used, the  $E_o$  and  $I_o^{\max} = F_d / T$  would be larger for the same maximum fluence of  $F_d \approx 1$  J/cm<sup>2</sup>. The maximum intensity from  $N$  laser pulses would scale as  $I^{\max} \propto 1/T$ , according to Eq. (17). The scaling implies that shorter pulses produce higher intensity. Currently, few-cycle laser pulses with a few fs duration have been synthesized. For transform limited pulse  $\Delta \nu = c(\lambda_1^{-1} - \lambda_2^{-1}) = 1/T$ , the reflective wavelength window of the mirror must be at least

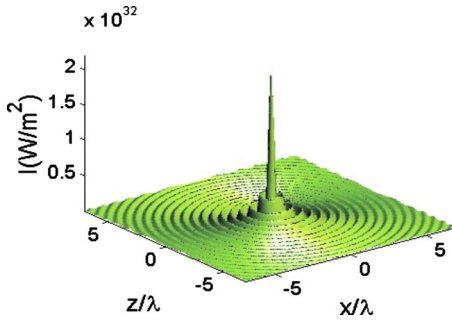


FIG. 3. (Color online) Intensity distribution for linear polarization showing peak intensity  $2.2 \times 10^{17} \text{ W m}^{-2}$ . We use  $N=N_o \approx 63$ ,  $w_0=1 \text{ m}$ ,  $I_o^{\text{max}}=10^{17} \text{ W m}^{-2}$  for single beam corresponding to  $T=100 \text{ fs}$  and fluence  $F_d=1 \text{ J cm}^{-2}$ , with  $\lambda=840 \text{ nm}$ ,  $R_m=20 \text{ m}$ , and  $R=1.1R_m$ .

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{\lambda_1^2}{cT - \lambda_1} = 2[\sqrt{(cT)^2 + \lambda^2} - cT], \quad (19)$$

where  $\lambda=(\lambda_2+\lambda_1)/2(=840 \text{ nm})$  is used to get the second equation, and  $T>\lambda_1/c$  in the first implies that the pulse duration cannot be smaller than the period of the highest frequency component.

By using pulses with  $T=1 \text{ fs}$  duration, the resulting intensity can be boosted a hundred times, up to  $I^{\text{max}} \approx 10^{34} \text{ W m}^{-2}$ . This requires broadband mirror  $\Delta\lambda=1184 \text{ nm}$  (according to Eq. (20)) that can tolerate fluence of at least  $F_d \approx 1 \text{ J/cm}^2$ . Such mirror is close to that of Takada *et al.*<sup>16</sup> who demonstrated high threshold  $F_d > 1 \text{ J/cm}^2$  hafnia/silica multilayer chirped mirror, with reflective window of  $\Delta\lambda=300 \text{ nm}$ , corresponding to pulse bandwidth of  $2\pi \times 10^{14} \text{ s}^{-1}$  and duration of around  $10 \text{ fs}$ . Double-chirped mirror<sup>20</sup> has been used for  $5.7 \text{ fs}$  pulses while metal-dielectric (Hf-SiO<sub>2</sub>) mirror<sup>21</sup> has threshold beyond  $10 \text{ J/cm}^2$ . Combination of both could produce the required mirror with threshold exceeding  $10 \text{ J/cm}^2$  for fs-pulses. For circularly polarized lasers, the results are shown in Fig. 4. The peak intensity  $I_y^{\text{cir}}$ , for y-component electric field is half of the peak intensity for linearly polarized case  $I^{\text{lin}}$  in Fig. 3. The remaining half is divided between the x- and y-components ( $I_x^{\text{cir}}$  and  $I_z^{\text{cir}}$ ), with energy predominantly at the twin peaks and the “side petals”, satisfying the expression

$$I^{\text{lin}} \approx I_y^{\text{cir}} + \sqrt{2}(I_x^{\text{cir}} + I_z^{\text{cir}}). \quad (20)$$

## VI. DISCUSSIONS

We have shown that multiple laser pulses reflected by a circular mirror can produce extremely intense laser field within a diffraction limited spot. The maximum intensity is unlimited, in principle. Laser dimension with giant proportion would create an awesome intensity toward and beyond the QED regime.

Although practical issue is not the focus of this work, we may briefly outline some of the issues. We have considered the effect of imperfect synchronization due to jitters and noise in optical components. It has been modeled stochastically through  $\delta_n = \Delta t \times \text{random}(0, 1)$  as fluctuations in the pulse arrival time. For an error within one optical pulse period  $\Delta t = T$ , the intensity is reduced only by a factor of two. The reduction factor increases with  $\Delta t$ . In the limit of large  $\Delta t$ , the enhancement approaches  $N$ —due to incoherent sum.

The multiple pulses are simultaneously released for obtaining coherent superposition at the focal point C. Using longer pulses would ease the synchronization. There are several existing schemes that can synchronize the laser pulses. One method involves  $n$  stages of interferometers to combine  $2^n$  beams.<sup>22</sup> Advanced synchronization mechanisms using systems of laser oscillators, radio frequency, and electronics have been developed in the NIF and Vulcan laser facilities.<sup>23</sup> Synchronization scheme for x-ray FEL at FLASH facility with jitter range of around  $50 \text{ fs}$ <sup>24</sup> may also be applicable here. An electro-optical technique to accomplish phase controlled beam combination by stimulated Brillouin scattering has been proposed recently.<sup>25</sup> These technologies can be tailored to synchronize the multiple laser pulses in this scheme.

In conclusion, we have proposed and analyzed a laser amplification scheme based on multiple  $100 \text{ fs}$  lasers in circular geometry with circular mirror that can scale up the intensity of laser field far beyond the current record, and beyond  $10^{29} \text{ W/cm}^2$  (the QED region). Higher intensity, up to  $10^{31} \text{ W/cm}^2$ , can be realized by using few-fs laser pulses with broadband ring mirror. Expressions for the intensity amplification factor and maximum intensity have been derived.

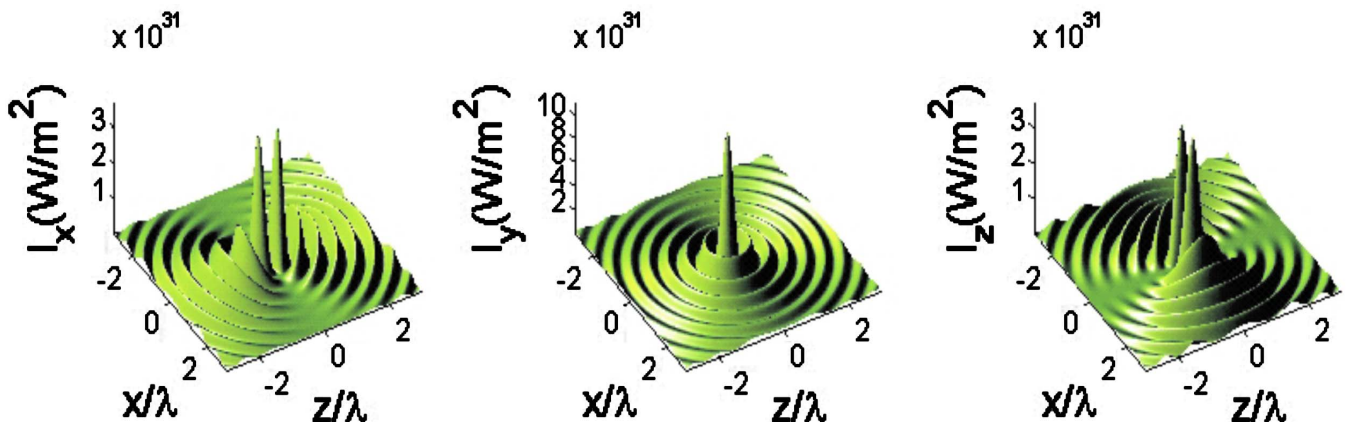


FIG. 4. (Color online) Intensity distributions for circularly polarized lasers. The peak intensities are smaller than the linear polarized case. Other parameters are the same as in Fig. 3.

Criteria for attaining superintense field are identified. We have shown that multiple laser pulses reflected by a circular mirror can produce, in principle, unlimited laser field intensity within a diffraction limited spot. The results show promising possibility for developing giant laser facilities capable of producing awesome laser intensities. The realization of such laser scheme is expected to have positive impact on the progress of laser fusion for sustainable energy, new science in nonlinear QED, and the application of superlasers for disintegrating threatening space rock en route to earth.

## ACKNOWLEDGMENTS

The author thanks Monash University for support through an internal grant, and Professor Kirk McDonald for discussions on paraxial corrections.

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