An Iterative Incremental Learning Algorithm for Complex-Valued Hopfield Associative Memory

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Abstract. This paper discusses a complex-valued Hopfield associative memory with an iterative incremental learning algorithm. The mathematical proofs derive that the weight matrix is approximated as a weight matrix by the complex-valued pseudo inverse algorithm. Furthermore, the minimum number of iterations for the learning sequence is defined with maintaining the network stability. From the result of simulation experiment in terms of memory capacity and noise tolerance, the proposed model has the superior ability than the model with a complex-valued pseudo inverse learning algorithm.

Keywords: Associative memory · Complex-valued model · Incremental learning

1 Introduction

In the past decades, numerous types of "Artificial Intelligence" models have been proposed in the field of computer science in order to realize the human-like abilities based on the analysis and modeling of essential functions of a biological neuron and its complicated networks in a computer. It has been noted that one of the interesting and challenging subjects is the imitation of memory function of the human brain. In the past decades, several types of artificial associative memory models and its improvements have introduced such as Hopfield Associative Memory (HAM) [7], and Bi-/Multi-directional Associative Memory (BAM, MAM) [6,11]. However, the majority of models are limited as offline and one-shot learning rules. Storkey and Valabregue [14] proposed an incremental learning algorithm for HAM based on the Hebb/Anti-Hebb learning. Chartier and Boukadoum [3] also proposed and analyzed the model with a self-convergent iterative learning rule based on the Hebb/anti-Hebb approach, and a nonlinear output function Diederich and Opper [4] proposed a Widrow-Hoff type learning. However, the convergence of these models is quite slow when the memory patterns are correlated. The local iterative learning, on the other hand, which is proposed by Blatt and Vergini [2] can be dealt with above kind of problems.

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Moreover, this model is able to define the minimum number of learning iterations with maintaining a network stability.

In regard as events in the real world, information representation using binary or bipolar state is insufficient. Jankowski et al. [9] have introduced complex-valued Hopfield Associative Memory (CHAM) with neurons processing a complex-valued discrete activation function. Conventionally, numerous studies that improve complex-valued models are introduced based on the improvements for real-valued models. Lee [13] applied a complex-valued projection matrix to CHAM (PInvCHAM) and analyzed the stability of the model by using energy function. The learning algorithm of these models, however, are characterized by a batch learning. Isokawa et al. [8] analyzed the stability of complex-valued Hopfield model with from a local iterative learning scheme viewpoint though the learning algorithm is Hebb learning base. In this paper, we introduce a local iterative incremental learning with CHAM based on Blatt and Vergini learning algorithm [2]. Here, we shall call the proposed model as BVCHAM. The noteworthy features of BVCHAM are described as follows; (i) the resultant of weight matrix in BVCHAM is approximated to the projection matrix operation. The network is able to store the patterns though the number of stored patterns p is larger than the number of neurons N (It is known that the memory capacity is limited as $p < N$ with a projection matrix learning model.), and (ii) the learning algorithm is guaranteed to calculate a weight matrix within a finite number of iterations with keeping a network stability.

The paper is divided as follows; Sect. 2 describes the dynamics of BVCHAM. Section 3 presents the network stability analysis for BVCHAM. In Sect. 4, it will be presented simulation experiments of BVCHAM in terms of the memory capacity and noise tolerance comparing with PinvCHAM. Concluding remarks are presented in Sect. 5.

2 Dynamics of a Local Iterative Learning for CHAM

This section presents the dynamics of a proposed model, we shall call as BVCHAM.

Let us suppose that the model stores complex-valued fundamental memory vectors X^p, where $X^p = [x_1^p, x_2^p, \ldots, x_N^p]^T$, N denotes the number of neurons, and p denotes the number of patterns. The components x_i^p are defined as follows;

$$x_i^p \in \exp\left[j2\pi n/q\right]_{n=0}^{q-1}, \quad i = 1, 2, \ldots, N \tag{1}$$

where q denotes a quantization value on the complex valued unit circle.

Here, supposing a new pattern X^l stores to the network by updating the weight matrix $W^{(l-1)}$ that already has $(l-1)$ patterns embedded. $X^{(l)}$ is presented n_l times to the network as follows;

$$W^{(l)} = W^{(l-1)} + \sum_{d=1}^{n_l} \Delta W_d^{(l)} \tag{2}$$

where, ΔW is defined as a following;

$$\Delta W_d^{(l)} = k^{d-1} \left(x_i^l - h_i \right) \left(x_j^l - h_j \right) / N. \tag{3}$$

The local field h_i which changes $(d-1)$ times is performed as a following;

$$h_i = \sum_{j=1}^{N} \left[W_{ij}^{(l-1)} + \sum_{r=1}^{d-1} \left(W_r^{(l)} \right)_{ij} \right] x_j^l. \tag{4}$$

Similar with Eq. (4), the local field h_j is defined. Here, the minimum number of iterations n_l can be defined as a following;

$$n_l \geq \log_k \left[N / \left(\pi/q - T \right)^2 \right] \tag{5}$$

where, a parameter k, called memory coefficient, is a real number that belongs to the interval $0 < k \leq 4$, and a parameter T is set between $0 \leq T < \pi/q$. It allows to perform in order to achieve aligned local fields with values of at least T [2]. q denotes a quantization value on the complex unit circle, which is described in a following part. The iterative weight update is performed until satisfying the criterion as $T >$ error ϵ, namely;

$$T > \epsilon = \sum_{i=1}^{N} \left\| \arg \left(\frac{h_i}{X_i^l} \right) \right\|. \tag{6}$$

In summary, the process of weight update is described as Algorithm 1. For the association process, the self-connections eliminated weight matrix $W^{'}$ is utilized. On the other hand, the weight matrix W which is calculated by Algorithm 1 is applied to the further incremental learning process.

Algorithm 1. An algorithm for a weight matrix $W^{(l)}$

Require: weight matrix $W^{(l-1)}$, fundamental memory vector X^l, error parameter T, memory coefficient k
Ensure: weight matrix $W^{(l)}$
  if $l = 1$ **then**
    Initialize W as a zero matrix
  end if
  Set $d = 1$
  Calculate an error ϵ using Eq. (6)
  while $d \geq n_l$ and $\epsilon > T$ **do**
    Calculate a weight matrix $W^{(l)}$ using Eqs. (2) and (3)
    Calculate an error ϵ using Eq. (6)
    Set $d \leftarrow d + 1$
  end while

The activation function is a complex projection function that operates on each component of the state vector as a following;

$$\phi(Z) = \begin{cases} \exp(j2\pi n/q), & \text{If } \left| \arg \left\{ \frac{Z}{\exp(j2\pi n/q)} \right\} \right| < \pi/q \text{ and } Z \neq 0 \\ \text{previous state}, & \text{If } Z = 0 \end{cases} \tag{7}$$

where, $\arg(\alpha)$ denotes the phase angle of α which is taken to range over $(-\pi, \pi)$. q denotes a quantization value on the complex unit circle, n takes an integer. We utilized a discrete complex unit circle model to determine recalled signals.

Thus, the dynamics of network is summarized as follows;

$$\begin{cases} h'_{(t)} = W' X_{(t)} & (8) \\ X_{(t+1)} = \phi\left(h'_{(t)}\right) & (9) \end{cases}$$

The stationary conditions for the recalled memory vector are described as a following;

$$0 \leq \arg\left(\frac{h'}{X^p}\right) < \frac{\pi}{q}. \quad (10)$$

3 Network Stability Analysis

In this section, based on Blatt and Vergini algorithm [2], the conditions of network stability in BVCHAM will be discussed, and it will be derived a criterion for the number of iterations n_l to guarantee the stability of X^l. Here, Dirac notation in $\mathbb{C}^N$ is utilized for simplifying the representation of proofs. Thus, Eqs. (3) and (4) can be described as follows;

$$\Delta W_d^l = \frac{k^{d-1}}{N} |X^l - h\rangle\langle X^l - h| \quad (11)$$

$$|h\rangle = \left[W^{(l-1)} + \sum_{r=1}^{d-1} \Delta W_r^l\right] |X^l\rangle \quad (12)$$

here, it satisfies $|X^l - h\rangle^\dagger = \langle X^l - h|$.

First of all, the changes of weight connection when a new memory vector is repeated to the network is analyzed. Here, $\Delta W_d^{(l)} (1 \leq d)$ is defined as a following;

$$\sum_{r=1}^{d} \Delta W_r^{(l)} = Q_d^{(l)} \frac{E^{(l-1)}|X^l\rangle\langle X^l|E^{(l-1)}}{N} \quad (13)$$

where,

$$E^{(l)} = I - W^{(l)} \quad (14)$$

here, I denotes an identity matrix which has same dimension with $W^{(l)}$.

It assumes that Eq. (13) holds for d-th iteration in order to prove $(d+1)$-th iteration. The difference of weight connection between the d-th and $(d+1)$-th iteration with a memory vector X^l and its local field h is expressed by Eqs. (12), (13) and (14), as follows;

$$|X^l - h\rangle = (1 - a^{(l)} Q_d^{(l)}) E^{(l-1)} |X^l\rangle \quad (15)$$

where,

$$a^{(l)} = \frac{\langle X^l|E^{(l-1)}|X^l\rangle}{N}. \tag{16}$$

The change in $(d+1)$-th iteration is expressed by Eqs. (11) and (15) as a following;

$$\Delta W_{d+1}^{(l)} = k^d \left(1 - a^{(l)} Q_d^{(l)}\right)^2 \frac{E^{(l-1)}|X^l\rangle\langle X^l|E^{(l-1)}}{N}. \tag{17}$$

Comparing with Eqs. (13) and (17), it can be regarded as a proportional relation with the same operator. Thus, the following recurrence relations are obtained;

$$Q_1^{(l)} = 1 \tag{18}$$

$$Q_d^{(l)} = Q_{d-1}^{(l)} + k^{d-1}\left(1 - a^{(l)} Q_{d-1}^{(l)}\right)^2. \tag{19}$$

From the Eqs. (13) and (14), Eq. (2) is represented as a following;

$$E^{(l)} = E^{(l-1)} - Q_{n_l}^{(l)} \frac{E^{(l-1)}|X^l\rangle\langle X^l|E^{(l-1)}}{N}. \tag{20}$$

Therefore, when a new memory vector is presented to the network, an operator which is proportional to the projector onto $E^{(l-1)}|X^l\rangle$ is added to the weight connection. According to Eq. (11), ΔW increases exponentially with the number of iterations d. However, if the following condition is satisfied, the elements of ΔW remain finite, i.e.;

$$0 \le \langle\phi|E^{(l)}|\phi\rangle \le \langle\phi|\phi\rangle \text{ for all } |\phi\rangle \in \mathbb{C}^N. \tag{21}$$

In the first learning step, it maintains $E^{(0)} = I$ and $Q_d^{(1)} = 1$. Furthermore, based on Cauchy–Schwarz inequality, Eq. (21) is verified for $l = 1$. Here, assuming that Eq. (21) is valid for l. Then, the eigenvalues of $E^{(l)}$ take between 0 to 1. Therefore, the following condition is satisfied;

$$\|E^{(l)}|\phi\rangle\|^2 \le \langle\phi|E^{(l)}|\phi\rangle. \tag{22}$$

Let us suppose that $a^{(l+1)} = 0$, $E^{(l)}|X^{l+1}\rangle = 0$ is derived from Eqs. (16) and (22), then $E^{(l+1)} = E^{(l)}$ is maintained from Eq. (20). Thus, Eq. (21) is also satisfied for $(l+1)$. In contrast, in case of $a^{(l+1)} \ne 0$, it is proved by the inductive hypothesis $0 < a^{(l+1)} \le 1$. Furthermore, according to [2], following conditions are maintained;

$$0 < a^{(l)} Q_d^{(l)} \le 1. \tag{23}$$

$$1 - a^{(l)} Q_d^{(l)} \le \left(\frac{k^{-d}}{a^{(l)}}\right). \tag{24}$$

From the following part, it focuses the memorized pattern $|X^\mu\rangle (1 \le \mu < l)$ and its local field $|h\rangle$ are able to be closer than other memorized patterns. The distance between $|X^\mu\rangle$ and $|h\rangle$ decreases exponentially with the number of iterations. This is derived by the induction process. First of all, the following condition is maintained by the Eqs. (14) and (22);

$$\left\| |X^\mu\rangle - W^{(l)}|X^\mu\rangle \right\|^2 \leq \langle X^\mu|E^{(l)}|X^\mu\rangle. \tag{25}$$

Here, Eq. (20) is generalized as a following;

$$E^{(l)} = E^{(\mu)} - \sum_{\nu=\mu+1}^{l} Q^{\nu}_{n_\nu} \frac{E^{(\nu-1)}|X^\nu\rangle\langle X^\nu|E^{(\nu-1)}}{N}. \tag{26}$$

Then, $\langle X^\mu|$ multiplied on the left hand side, and $|X^\mu\rangle$ multiplied on the right hand side to Eq. (26), therefore;

$$\langle X^\mu|E^{(l)}|X^\mu\rangle \leq \langle X^\mu|E^{(\mu)}|X^\mu\rangle. \tag{27}$$

For the Eq. (20) with a μ-th memory vector, $\langle X^\mu|$ is multiplied on the left hand side, and $|X^\mu\rangle$ is multiplied on the right hand side, i.e.;

$$\langle X^\mu|E^{(\mu)}|X^\mu\rangle = \langle X^\mu|E^{(\mu-1)}|X^\mu\rangle - Q^{(\mu)}_{n_\mu} \frac{(\langle X^\mu|E^{(\mu-1)}|X^\mu\rangle)^2}{N}. \tag{28}$$

Considering with Eq. (16), Eq. (28) is described as a following;

$$\langle X^\mu|E^{(\mu)}|X^\mu\rangle = Na^{(\mu)}\left(1 - a^{(\mu)}Q^{(\mu)}_{n_\mu}\right). \tag{29}$$

Furthermore, N and $a^{(\mu)}$ multiplied to Eq. (24) with a μ-th memory vector, i.e.;

$$Na^{(\mu)}\left(1 - a^{(\mu)}Q^{(\mu)}_{n_\mu}\right) \leq Nk^{-n_\mu}. \tag{30}$$

From Eqs. (25), (27), (29) and (30), the following condition is obtained;

$$\begin{aligned} \left\| |X^\mu\rangle - W^{(l)}|X^\mu\rangle \right\|^2 &\leq \langle X^\mu|E^{(l)}|X^\mu\rangle \leq \langle X^\mu|E^{(\mu)}|X^\mu\rangle \\ &= Na^{(\mu)}\left(1 - a^{(\mu)}Q^{(\mu)}_{n_\mu}\right) \leq Nk^{-n_\mu}. \end{aligned} \tag{31}$$

According to the stability condition as Eq. (10), Eq. (31) is described as a following;

$$\left\| |X^\mu\rangle - W^{(l)}|X^\mu\rangle \right\|^2 \leq Nk^{-n_\mu} \leq \left(\frac{\pi}{q} - T\right)^2 \tag{32}$$

where, q denotes a quantization value on the complex unit circle. The optimal stability can be controlled by the parameter T [5,12]. In addition, the network stability is guaranteed in case of $T = 0$. Furthermore, from Eq. (32), the minimum number of iterations n_l is derived as Eq. (5).

4 Simulation Experiment

This section presents the simulation experiments comparing with a pseudo inverse learning model (PInvCHAM) [1] and a proposed model (BVCHAM), in terms of memory capacity and noise tolerance.

4.1 Condition

Table 1 shows the simulation conditions to evaluate the memory capacity and the noise tolerance. In this experiment, the number of pairs p is increased from 1 to 220 at intervals of 10 ($p = 1, 10, 20, \ldots, 220$). Noise tolerance is a significant property for the associative memory. Typically, "noise" is roughly divided into two types in associative memory. One is the similarity of the stored patterns, another is the stored patterns contain noise itself. Due to the proposed model has the similar properties with a pseudo inverse learning model, it is expected that the proposed model is able to learn the correlated memory vectors without errors. In this paper, therefore, we consider about a latter type of noise which is contained in an initial input with 0 to 50 [%] by the salt & pepper noise. Furthermore, it is known that the ability of a complex-valued associative memory is dependent upon the number of divisions of a complex-valued unit circle. Here, we set 4, 8 and 16 divisions for this simulation.

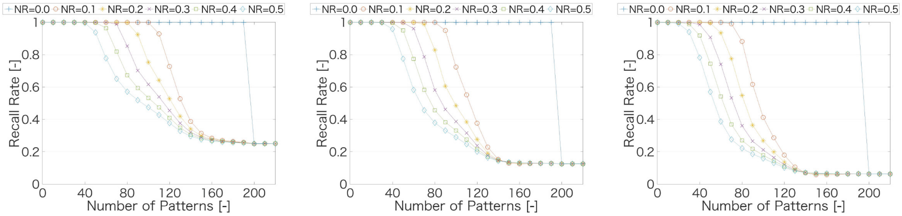
Table 1. Simulation condition.

Number of pairs p : 1–220
Number of neurons N: 200
Number of divisions q: 4, 8, 16
Data set configuration: Amplitude: 1.0, Phase: Rand

4.2 Result

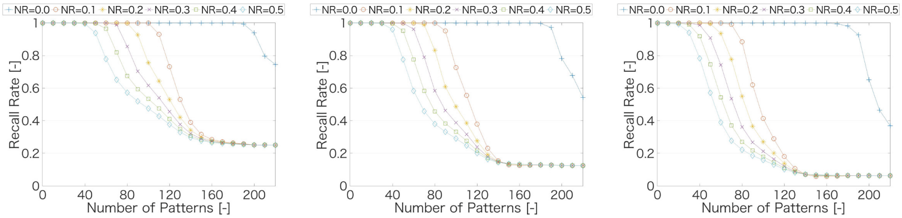
In Fig. 1, PInvCHAM maintains the high recall rate, especially $NR = 0.0$. However, due to the limitation of the pseudo inverse learning, the recall rate is suddenly dropped under the condition $N \leq p$. As shown in Fig. 2, due to the weight matrix of BVCHAM can be approximated as the weight matrix by a pseudo inverse learning, the recall rate of BVCHAM shows the similar or superior results (especially in Fig. 2(a)) than PInvCHAM. Here, it focuses on the results with $NR = 0.0$ in Fig. 2. Since the weight matrix of BVCHAM is not equal to the identity matrix at $p = N$, BVCHAM is able to maintain the high recall rate than PInvCHAM even if the condition is under $N \leq p$.

From the above results, it is regarded that BVCHAM has the following noteworthy advantages; although a proposed learning algorithm is incremental, the ability can be comparable to a batch learning as a pseudo inverse learning algorithm, and it is able to overcome the limitation of a pseudo inverse learning algorithm that is characterized by $p < N$.



(a) $q = 4$ (b) $q = 8$ (c) $q = 16$

Fig. 1. Results of recall ratio for a pseudo inverse learning model.



(a) $q = 4$ (b) $q = 8$ (c) $q = 16$

Fig. 2. Results of recall ratio for a BV incremental learning model.

5 Conclusion

This paper introduced a local iterative incremental learning algorithm for a complex-valued Hopfield associative memory. Furthermore, we presented the network stability analysis which derived the weight matrix of BVCHAM is approximated as a weight matrix from a complex-valued pseudo inverse learning algorithm, and the minimum number of iterations with maintaining a network convergence. From the result of simulation experiment in terms of memory capacity and noise tolerance, BVCHAM has the superior ability than the models with the Hebb learning and a complex-valued pseudo inverse learning algorithm. Noteworthy, unlike the model with a pseudo inverse learning algorithm, the proposed model is able to maintain the high recall rate even if the number of stored patterns is larger than the number of neurons.

As a future work, we will extend the proposed incremental learning algorithm to hetero-association models, such as bi/multi-directional associative memory models [10].

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