# The use of initial imperfection approach in design process and buckling failure evaluation of axially compressed composite cylindrical shells

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**ABSTRACT**

Thin-walled cylindrical shells are susceptible to buckling failures caused by the axial compressive loading. During the design process or the buckling failure evaluation of axially-compressed cylindrical shells, initial geometric and loading imperfections are of important parameters for the analyses. Therefore, the engineers/designers are expected to well understand the physical behaviours of shell buckling to prevent unexpected serious failure in structures. In particular, it is widely reported that no efficient guidelines for modelling imperfections in composite structures are available. Knowledge obtained from the relevant works is open for updates and highly sought. In this work, we study the influence of imperfections on the critical buckling of axially compressed cylindrical shells for different geometries and composite materials (Glass Fibre Reinforced Polymer (GFRP), Carbon Fibre Reinforced Polymer (CFRP)) and aluminium using the finite element (FE) analysis. Two different imperfection techniques called eigenmode-affine method and single perturbation load approach (SPLA) were adopted. Validations of the present results with the published experimental data were presented. The use of the SPLA for introducing an imperfection in axially compressed composite cylindrical shells seemed to be desirable in a preliminary design process and an investigation of a buckling failure. The knockdown factors produced by the SPLA were becoming attractive to account for uncertainties in the structure.

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## 1. Introduction

Thin-walled cylindrical structures have been widely used in many industrial applications. However, thin cylindrical shells are vulnerable to buckling failures caused by the induced compressive loading. Too thin cylindrical shells and larger design loads may make the structures more prone to buckling failure. Real case studies on buckling failures of pressurized vessels have been reported by Jones [1]. Next, Teng and Zhao [2] deployed state-of-the-art finite element analyses on the failed pressure vessels reported by Jones [1]. Teng and Zhao [2] examined the validity of the formulations for evaluating real vessels with geometric imperfections through a comparison of theoretical predictions with experimental results, which established the limited sensitivity of the buckling load to initial imperfections. Indeed, initial geometric and loading imperfections are of

important parameters for analyzing axially compressed cylindrical shells. Iwicki et al. [3] carried out a failure analysis on cylindrical silo shells by performing linear and non-linear buckling analyses with different initial geometric imperfections. To avert unexpected catastrophic failure of structures of which thin cylindrical shells are essential components, the designers and engineers shall acquire sufficient knowledge on the physical behaviours of shell buckling.

Nowadays, rapid developments in the industrial manufacturing technology encourage the engineers/designers to seek more suitable candidate materials that demonstrate excellent gains in weight, stiffness and stability. Advanced composite materials have been extensively developed to offer an outstanding ability in reducing the structural weight while increasing its strength. The composite materials are particularly desirable in the aerospace structures that have been known to have operational nature under compressive load during service. Various approaches have been adopted to predict the reasonable buckling load for shell structure through experimental works and finite element analyses. Since experimentations on composite materials are time consuming and require high cost and complex experimental setup, numerical modelling is always sought to support a design process.

In particular, with regard to the aerospace structures, prediction of knockdown factor $\rho$ remains highly dependent on the empirical guideline from NASA SP-8007 [4] that uses the conservative lower bound curve. Furthermore, the guideline is mainly intended to the isotropic materials, and less information corresponding to the composite material is described. The approach stated in the guideline excessively led to conservative designs on estimating cylinder buckling load as highlighted by researchers [5–8]. The NASA SP-8007 [4] also does not take into account of the full potential of materials on knockdown factor [9]. In particular, Arbelo et al. [10] reported the use of NASA SP-8007 [4] in the design of the European Ariane-5 rocket launcher. Even though the rocket has an outstanding performance, the structural design seemed to be considerably conservative, resulting in high costs in the production and operation [11,12]. The EU project DESICOS has been launched in 2012 and is underway to propose a new design guideline for designing imperfection in composite launcher structures [13]. A new efficient design approach is expected to come up with possibilities for reducing the design cost and structural weight.

The eigenmode-affine method is one of the techniques for introducing imperfections on the structural model subjected to compressive loading. The method that uses the eigenmode shapes together with specifying scaling factor of the imperfection amplitude has been extensively employed by researchers [3,11,12,14–18]. However, the range of the scaling factor applied in the eigenmode would often be on trials. Another famous approach for evaluating post-buckling problems, the arc-length method, has been developed in 1970s. However, the method failed to predict the stable post-buckling state after the global instability as recently reported by Casado et al. [19].

The single perturbation load approach (SPLA) which was proposed by Hühne et al. [8] is relatively a new method for designing an imperfection on axially compressed shell structures. The method uses the influence of a single laterally applied load to the surface of the model in order to simulate the worst geometrical imperfection of typical structural models such as cylindrical shells. The applied lateral load would produce a local-dimple that acts as the imperfection amplitude. By varying the amplitudes during the compression loading process, the local-dimple imperfection may then trigger local and global instabilities to the structure. Arbelo et al. [10] reported several studies on estimating the cylinder knockdown factor using SPLA method. Although significant cylinder knockdown factor data using SPLA have been produced, there are not sufficient information on some engineering materials with the variation of cylindrical geometries such as different aspect ratios of thickness, radius and length ($L/r$ and $r/t$). For instance, the application of SPLA in Glass Fibre Reinforced Polymer has not been extensively explored in comparison to that in Carbon Fibre Reinforced Polymer.

This paper presents the responses of two different imperfections (by the eigenmode-affine method and SPLA) on the cylindrical shells for different geometries and materials. The results obtained by using the eigenmode-affine and SPLA are then validated with the published experimental data. The findings presented here would show the more suitable approach for the use in the design process and buckling failure evaluations.

## 2. Materials and method

Different cylinders geometries, aspect ratios and material properties are investigated with the aid of FE analyses: linear eigenvalue and nonlinear static analyses.

### 2.1. Finite element modelling and material properties

After performing convergence tests on the number of elements used in the CFRP cylindrical shell models (see Table 1), about 11,660 four-node doubly curved shell elements with reduced integration and hourglass control (S4R) were found to be suitable to model the composite cylinders. It was reported that the imperfection in the CFRP cylindrical shells is more sensitive than that in the GFRP material [20]. In this present work, the use of more than 10,000 elements seems to result in insignificant difference in buckling loads. All the analyses were carried by using a finite element software package of ABAQUS V6.10. The schematic of the axially compressed cylindrical shell is shown in Fig. 1. As shown in Fig. 1, displacement and rotation constraints are applied on one end of the cylindrical shell, and the load-controlled displacement $\Delta U$ with slow quasi-static compression is applied on the other end to infuse instability of the cylinder.

**Table 1**
Convergence study on the CFRP cylindrical shells.

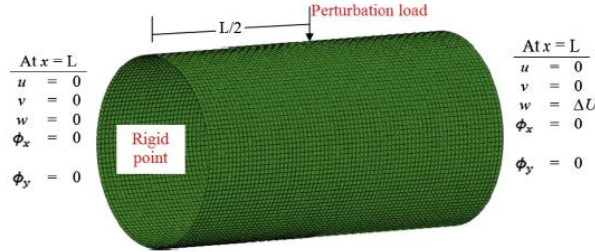| Number of elements | | | | |
|---|---|---|---|---|
| 2860 | 4998 | 6591 | 10,241 | 11,660 |
| Buckling load (kN) | | | | |
| 279.89 | 261.62 | 255.74 | 249.6 | 248 |



**Fig. 1.** Boundary condition of FE modelling by SPLA.

**Table 2**
Cylindrical shell geometries used in FE models.

| Authors | Unit | Bisagni [20] | Englitis [17] |
|---|---|---|---|
| Cylinder designation | | CFRP | GFRP |
| Radius, $r$ | (mm) | 350 | 150 |
| Free length, $L$ | (mm) | 520 | 560 |
| Nominal thickness, $t$ | (mm) | 1.32 | 1.1 |
| $r/t$ | | 265 | 136 |
| $L/r$ | | 1.45 | 3.73 |
| Controlled displacement, $\Delta U$ | (mm) | 2 | 2 |

**Table 3**
Material properties of CFRP and GFRP.

| Stiffness | Unit | CFRP [20] | GFRP [17] |
|---|---|---|---|
| Elastic modulus $E_{11}$ | (N/mm$^2$) | 52,000 | 18,660 |
| Elastic modulus $E_{22}$ | (N/mm$^2$) | 52,000 | 18,280 |
| Shear modulus $G_{12}$ | (N/mm$^2$) | 2350 | 4560 |
| Poission's ratio $\upsilon$ | | 0.302 | 0.16 |
| Density $\rho$ | (kg/mm$^3$) | $1.32 \times 10^{-8}$ | – |
| Ply thickness $t_{ply}$ | (mm) | 0.33 | 0.275 |

The cylinder geometries used in this study referring to those experimentally done by Eglitis [17] for the GFRP laminated shell and by Bisagni [20] for the CFRP laminated shell, and the details are presented in Table 2. Similarly, material properties of the monolithic laminates of carbon and glass fibre composites reported by Eglitis [17] and Bisagni [20] were also used in this study and presented in Table 3. The layers of the cylindrical shell consist of four-ply with stacking sequences of [0/45/−45/0] for CFRP laminated shell and unidirectional-ply of [0/0/0/0] for GFRP laminated shell.

### 2.2. Eigenmode-affine method

In this method, the FE linear buckling analyses were first carried out to obtain the eigenvalues and eigenmodes of the cylindrical shells. Next, the initial geometrical imperfection based on the first eigenmode shape was introduced on the cylindrical shell. The first eigenvalue was used to calculate the critical perturbation load $P_{cr,eigen}$. The range of the imperfection magnitude $a$ was investigated by varying the ratio of $a/t$, where $t$ is the shell thickness.

### 2.3. Single perturbation load approach

In general, the procedure of the SPLA in the finite element analysis for axially compressed cylindrical shell models has three loading steps [8]. In the first step, a lateral perturbation load $P_0$ is applied at the centre of the cylindrical shell (as

illustrated in Fig. 1). It is purposely used to produce a single buckle as the worst imperfection mode until the equilibrium state of the shell is reached. Next, in the second step, the cylindrical shell is loaded by quasi-static axial compression until the first instability point is reached. The Newton–Raphson algorithm with adaptive time increments is adopted/chosen in both steps. In the third step, a uniform end-shortening displacement is introduced to act as the axial compressive load. At this stage, the dynamic responses of load-end shortenings are taken care of by employing ABAQUS/Explicit that is a spe-cially-dedicated module of finite element analyzer to solve highly nonlinear systems.

## 3. Results and discussion

Prior to performing response analyses on the cylindrical shells, the sensitivity study was conducted to determine a suit-able value for the artificial damping factor. It is known that too small value of the artificial damping would possibly result in a singularity of the tangent stiffness matrix. Meanwhile, a large artificial damping value would lead to over-damped results. The stacking design of unidirectional-ply of [0/0/0/0] is chosen to perform this sensitivity study. This type of stacking sequence was also chosen by Castro et al. [12] for finding the suitable artificial damping factor. The responses of different artificial damping values for the GFRP cylindrical shells are presented in Fig. 2a. It can be observed that the use of the damping factors $c = 4e-9$ and $4e-8$ fail to reach the first local snap-through. This feature is associated with the singularity of the tangent stiffness matrix, resulting in not convergence problem. Meanwhile, a larger artificial damping value $c = 4e-5$ seems to produce over-damped results. Thus, the artificial damping value $c = 4e-7$ that produces a convergence result is chosen in this study.

The knockdown factors ($= P_{cr}/P_{cr,perfect}$) versus variations of the imperfection amplitude over the thickness $a/t$ obtained by using the eigenmode-affine method are presented in Fig. 2b. The perturbation load $P_{cr,perfect}$ was obtained from the nonlinear FE analysis on the perfect cylindrical shell. The imperfection sensitivities can be noticed from the significant drop of the knockdown factors over the normalized values of $a/t$. Noticeable reductions of the knockdown factor are observed in both CFRP and GFRP cylinders for small imperfection magnitudes (up to $a/t \approx 0.3$). Meanwhile, the knockdown factors become
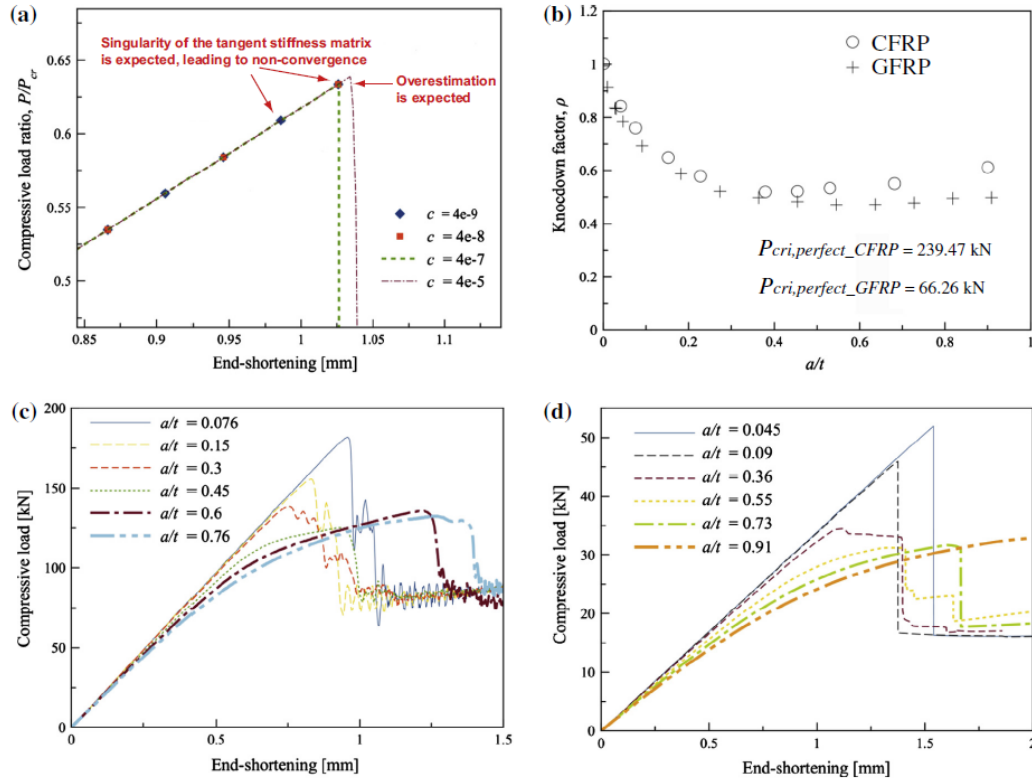


Fig. 2. (a) Convergence analysis for GFRP cylindrical shell; (b) Distribution knockdown factors by the eigenmode-affine method; Variations of imperfection amplitude by eigenmode-affine: (c) CFRP cylinder and (d) GFRP cylinder.

relatively stable towards the lowest value before ascending with different rate. The lowest knockdown factors $\rho$ of the CFRP and GFRP cylinders are found to be around 0.52 and 0.48, respectively. For different imperfection amplitudes as plotted in Fig. 2c and d, the CFRP and GFRP cylindrical shells show variations of the response of the load-end shortening. Based on the findings shown in Fig. 2b–d, the eigenmode-affine method requires significant efforts to justify the suitable imperfection magnitude to be introduced in the cylindrical models, especially with different material properties, ply arrangement and geometry.

Fig. 3a and b show the responses of the knockdown factor for the CFRP and GFRP cylinders using SPLA in the function of perturbation loads and normalized imperfection amplitudes. Stable horizontal responses at the lower bound are observed in both materials. The initial stable horizontal response is well known called the minimum/threshold perturbation load $P_{min}$. A perturbation load larger than $P_{min}$ will be about constant. The perturbation loads smaller than the minimum/threshold perturbation load $P_{min}$ trigger instability to the cylindrical shell before resulting in natural buckling wave patterns. The perturbation load larger than $P_{min}$ would act as a worst imperfection that directly leads to the cylinder load carrying capacity. There is a typical method that has been adopted by researchers as reported by Castro et al. [12] for determining the normalized knockdown factor. The normalized knockdown factor can be obtained by making the ratio of the lower bound value over the top bound value. Fig. 3 reveals that, the normalized knockdown factors for the CFRP and GFRP cylinder are found to be around 0.67 and 0.63, respectively. The corresponding minimum/threshold perturbation loads $P_1$ were found to be around 60 kN for CFRP cylinder and around 35 kN for GFRP cylinder. Meanwhile, for the given $P_{cr,perfect}$, the maximum load carrying capacity of the CFRP ($P_{cr,perfect}$ = 239.47 kN) and GFRP ($P_{cr,perfect}$ = 65.78 kN) cylinders were found to be around of 130 kN and 42 kN, respectively.

To demonstrate the ability of the methods, the results obtained by using the eigenmode-affine method and SPLA are compared with the published data [17,20] as presented in Table 4. It can be seen from Table 4, the critical buckling loads for both CFRP and GRFP cylinders predicted by using the eigenmode-affine method show lower values than the experimental data and those estimated by SPLA. It simply states that the eigenmode-affine method would give more conservative estimations
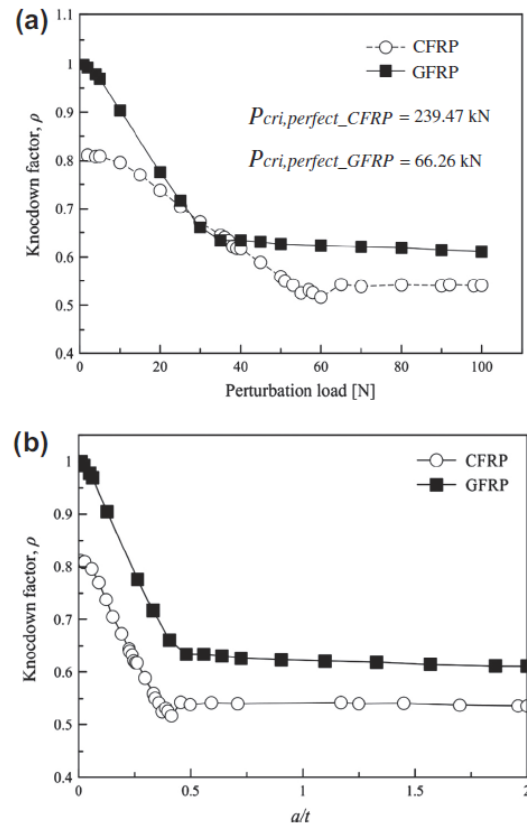


Fig. 3. Distribution knockdown factors by SPLA: (a) Perturbation loads; (b) Normalized imperfection amplitude $a/t$.

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