Application of Correlation as a Measure of Performance

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***Abstract*— Correlation generally shows the relationship between variables. A judicious use of this relationship may yield a measure of performance for a given algorithm. In this study the correlation measure $R_p^{\ 2}$ derived from the Unreplicated Linear Functional relationship (ULFR) model will be shown to be a useful measure of performance in selected procedure or algorithm for a particular image registration method, a medical treatment, a character recognition method, and a compression method. The main result of these numerical studies strongly suggests that $R_p^{\ 2}$ is potentially useful as a performance measure in a wide range of imaging problem.**

I. INTRODUCTION

Valuable information may be obtained by comparing a transformed image $\{b_{jk}; j=1,\cdots,n, k=1,\cdots m\}$ with its original form $\{a_{jk}; j=1,\cdots,n, k=1,\cdots m\}$ .Comparisons can be carried out when *p*-summary statistics $x_i=\left(x_{1i},x_{2i},\cdots,x_{pi}\right)'$ from $\{a_{ij}\}$ is compared with $y_i=\left(y_{1i},y_{2i},\cdots,y_{pi}\right)'$ from $\{b_{ij}\}$.

Consider the situation where both x_i and y_i represent the fixed and unobservable true vectors X_i and Y_i that were subject to error, in particular

$$\left.\begin{array}{l} x_i = X_i+\delta_i \\ y_i = Y_i+\varepsilon_i \end{array}\right\} i=1,\ldots,n \qquad (1)$$

where $\delta_i=\left(\delta_{1i},\delta_{2i},\ldots,\delta_{pi}\right)'$ and $\varepsilon_i=\left(\varepsilon_{1i},\varepsilon_{2i},\ldots,\varepsilon_{pi}\right)'$.

Both x_i and y_i can be observed in such a way that they are from two independent processes, especially in image processing. Assume both error vectors are mutually and independently normally distributed.

Further, suppose that $Y_i=\left(Y_{1i},Y_{2i},\ldots,Y_{pi}\right)'$ and $X_i=\left(X_{1i},X_{2i},\ldots,X_{pi}\right)'$ are linearly related such that

$$Y_i=\alpha+\beta X_i,\ i=1,\ldots,n \qquad (2)$$

Both equation (1) and equation (2) make up the Unreplicated Linear Functional relationship (UFLR) model with single slope, [1] - [3].

Given the ULFR model with single slope defined by Equations (1) and (2), the maximum likelihood estimators of α , β , X_i and σ_k^2 are

$$\hat{\alpha}=\bar{y}-\hat{\beta}\bar{x}$$

$$\hat{\beta}=\frac{-\left(\lambda S_{xx}-S_{yy}\right)+\sqrt{\left(\lambda S_{xx}-S_{yy}\right)^2+4\lambda S_{xy}^2}}{2S_{xy}}$$

$$\hat{X}_i=\frac{\lambda x_i+\hat{\beta}\left(y_i-\hat{\alpha}\right)}{\lambda+\hat{\beta}^2}$$

and $\hat{\sigma}^2=\frac{1}{n-2}\left\{\sum_{i=1}^{n}\left(x_i-\hat{X}_i\right)'\left(x_i-\hat{X}_i\right)\right.$
$\left.+\frac{1}{\lambda}\sum_{i=1}^{n}\left(y_i-\hat{\alpha}-\hat{\beta}\hat{X}_i\right)'\left(y_i-\hat{\alpha}-\hat{\beta}\hat{X}_i\right)\right\}$

where λ is the ratio of error variances, and

$$S_{xx}=\sum_{i=1}^{n}x_i'x_i-n\bar{x}'\bar{x}\,,\ S_{yy}=\sum_{i=1}^{n}y_i'y_i-n\bar{y}'\bar{y} \text{ and}$$

$$S_{xy}=\sum_{i=1}^{n}x_i'y_i-n\bar{x}'\bar{y}\,.$$

Finally, the correlation measure derived from the ULFR model is

$$R_p^2=\frac{\hat{\beta}S_{xy}}{S_{yy}} \text{ when } \lambda=1. \qquad (3)$$

In the image registration problem the δ_i and ε_i could be errors due to different body posture for a given patient when the chest X-ray image was being taken at two time points.

In the compression problem, the non-availability of a full reference image results in the existence of δ_i . The use of a lossy compression method may be the cause of ε_i .

For the Chinese character recognition problem, δ_i could be the consequences of different writing style and ε_i could be due to the choice or estimation of feature vector in the discrimination process.

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II. N- Control Point Registration

Let $A(j)^{(k)} = (x_j^{\,k}, y_j^{\,k})$, $j = 1, \cdots, N$ represent n-points (landmarks) on the ROI from the kth – image. Without loss of generality, let $d_{12}(j)$ be the Euclidean distance between $A(j)^{(1)}$ and $A(j)^{(2)}$, that is the corresponding point-j for image 1 and image 2. Further define $V_{xj} = x_j^1 - x_j^2$ and V_x = median $[V_{x1}, V_{x2}, \cdots, V_{xN}]$. Similarly define the median vertical shift as

$$V_y = \text{median}\ [y_1^1 - y_1^2, y_2^1 - y_2^2, \cdots, y_N^1 - y_N^2].$$

The N-control point registration is performed by treating the first image as a reference image and the second image is subject to a vertical displacement V_y and a horizontal displacement V_x such that $d_{12}(j)$, $j = 1, \cdots, N$ are minimized.

Figure 1(a), 1(b) and 1(c) shows a plaque located at a distance of 72 cm, 62 cm and 52 cm respectively from a fixed camera. Figure 1(d), 1(e) and 1(f) are the cropped image of Figure 1(a), 1(b) and 1(c), respectively with the same image size (574 x 488) that capture the object of interest (plaque).

Figure 2 (a), 2 (b) and 2 (c) are the cropped image from Figure 1 (a), 1 (b) and 1 (c) that capture the whole plaque as the object of interest. These images are then subjected to 4 control point registration procedure and then resized using the MATLAB command

$$\text{MAKETFORM ['affine', } \begin{pmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{]}$$

where for example in Figure 1, $C_x = \dfrac{2500}{2800} \approx 0.9$. and $C_y = \dfrac{2048}{2048} = 1$. C_x and C_y is the required percentage for resizing. The program will then interpolate the new pixel value using the bilinear interpolation. Figure 3 shows the resultant image of 574 x 488 pixels.

Table 1 compares the similarity measures for images from Figure 1(d), 1(e) and 1(f) when no image registration and resizing were performed. In all cases, images were highly dissimilar. Table 2 compares the similarity measures for images from Figure 3(a), 3(b) and 3(c) when image registration and resizing were performed. Comparing Table 1 and Table 2, the result shows that R_p^2 is more accurate as a measure of performance of the proposed image registration method (inclusive of resizing).

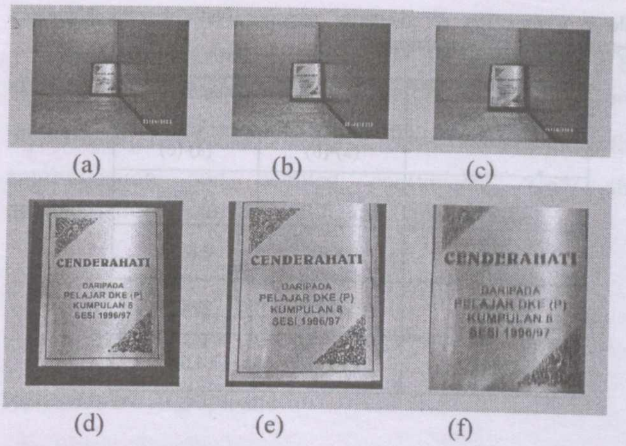


Fig. 1 One object taken at three different distance using the same camera that produce the same image size of 1944 × 2592, (a) Camera distance:72 cm, (b) Camera distance:62 cm, (c) Camera distance:52 cm. (d), (e) and (f) is the image of region of interest cropped with image size of 574 × 488.

(a) 574 × 488 (b) 675 × 563 (c) 808 × 683

Fig. 2 Sub image from Fig. 2 (a), Fig. 2(b) and Fig.2(c) respectively after 4 control point registration. Note the different image size though the region of interest is captured.

(a) (b) (c)

Fig. 3 The resultant image from Fig. 2 after resizing with Affine transformation. All images has the same size of 574×488 pixels.

Table 1 Comparison of similarity measures (No resizing and no translation)

Similarity measures	Comparing Images (Fig. 1) (d)-(e)	(d)-(f)
R_p^2	0.1350	0.0499
R_S^2	0.0747	0.0376
MSE	8213.0	8093.2
PSNR	8.9858	9.0496

Table 2 Comparison of similarity measure (with 4 control points registration and Affine resizing method)

Similarity measures	Comparing Images (Fig. 3)	
	(a)-(b)	(a)-(c)
R_p^2	0.7191	0.7905
R_S^2	0.5592	0.6754
MSE	3103.7	2257.4
PSNR	13.2120	14.5947

Other measures of similarity considered were,

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\| I(i,j) - K(i,j)\|^2$$

where I is second visit image and K is the first visit image,

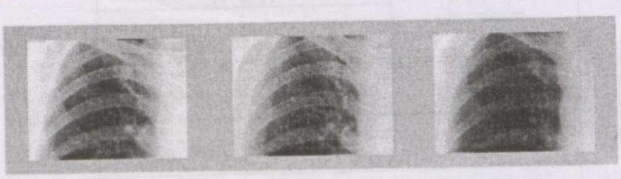
and $$PSNR = 20\times\log_{10}\left(\frac{MAX}{\sqrt{MSE}}\right)$$

where $MAX = 2^B - 1$, B = bit. The digital image is coded using 12 bit DICOM format, therefore $MAX = 2^{12} - 1 = 4095$.

Finally R_S^2 is the coefficient of determination for simple linear regression.

III. Using R_P^2 for Measuring Performance of a Medical Treatment

The success of treatment for pulmonary tuberculosis (PTB) patients is frequently determined from the comparison of a series of chest radiographs taken at different time points [4]. Using the methods of image registration and resizing as explained in section II, Fig. 4 shows three images of the same patient taken at three different time points. Table 3 shows higher correlation for the 1st visit and 2nd visit images and a lower R_P^2 value for the 1st and last visit which may be use as an indicator of patient recovery. In particular, decreasing R_P^2 value may indicate the performance or success of patient's treatment in the sense that the infected region of interest is healing (clearing up).



(a)1st visit (b)2nd visit . (c) 3rd visit

Fig. 4 Subset image after image registration and resizing

Table 3: R_P^2 value for selected PTB area.

Patient	Selected PTB area	
	1st visit and 2nd visit	1st visit and last visit
1	0.91	0.86
2	0.91	0.89
3	0.89	0.90
4	0.94	0.95
5	0.86	0.90
6	0.69	0.71
7	0.84	0.86
8	0.94	0.91
9	0.95	0.93
10	0.97	0.93
11	0.94	0.94
12	0.83	0.90
13	0.85	0.83
14	0.93	0.84
15	0.95	0.93

IV. Character Recognition Problem

A. *Introduction*

A novel recognition algorithm [1] was applied on a database with 3000 frequently used Chinese characters. Handwritten characters comprising different writing style, Fig. 5 and Fig. 6, are to be identified after comparison with the characters in the database.

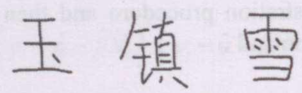


Fig. 5: A sample of 3 Chinese characters taken from writer A

Fig. 6: A sample of 3 Chinese characters taken from writer B

B. *The Experiment*

The recognition system was developed in a Dell Vostro 1400 N-Series notebook of Intel(R) Core(TM)2 Duo Processor T5470 and 1GB (2×512MB) 667MHz Dual Channel DDR2 SDRAM. The programming language used is MATLAB and the implementations only utilize one CPU core.

Two writers reproduce each of the 3000 characters in CL2009 only once. Each writer was given one week to complete the reproduction process under similar conditions. The first writer (A) has more than 15 years of experience with Chinese character, whereas the second writer (B) has 6 years of experience. The Wacom Intuos®3 pen tablet was used by each writer for the reproduction process. For each character, 128 points are used to represent each stroke. Thus, a w-strokes character, for example, will have a total of $128\times w$ points.

Once a character is written, it is cropped and normalized, and then converted into the X-graph and Y-graph (see Fig. 7), which in turn was subjected to the Haar wavelet transformation. The derived feature vector is subjected to a two-stage classification procedures; firstly the rough classification and secondly the fine classification.

The performance of the proposed recognition algorithm using R_p^2 is also studied with the city block distance with deviation (CBDD) [5], minimum distance (MD) [6], compound Mahalanobis function (CMF) [7] and modified quadratic discriminant function (MQDF) [8]. The experiment was carried out for both normalized and non-normalized characters.

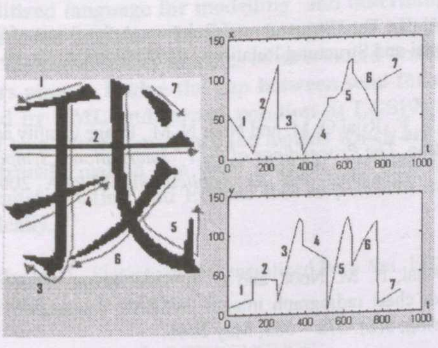


Fig. 7: X-graph (above) and Y-graph (below) of Chinese character '我' (means I or me)

C. X-graph and Y-graph

The X-graph is defined as $\{t, x_t\}$, $1\le t\le N = 128\times w$ where x_t is the value of the x-coordinate of a point on the character at position (space) t. The Y-graph $\{t, y_t\}$ is similarly defined. The subscript t in $\{t, x_t\}$ and $\{t, y_t\}$ depend on stroke direction, stroke order and stroke number, thus preventing the possibility of different pattern for the same character. The motivation for using these graphs lies in its properties of uniqueness and invariance, and that they are simple to use. As an example shown in Fig. 7, the values in the X-graph drop while in the Y-graph the values rise, when the first stroke is written; whereas in the case of the second (horizontal) stroke (this order is also fixed in Chinese character), the values in the X-graph rise while the corresponding values for the Y-graph remain unchanged. The process was repeated until the seventh stroke. Consequently, the feature vectors obtained are $\left[x_1,\dots x_N\right]^T$ and $\left[y_1,\dots y_N\right]^T$, $N = 7\times 128$ representing the X-graph and Y-graph respectively.

D. Haar wavelet transform

Haar wavelet reduces the size of the feature vector by creating two new sequences of points $\mathbf{a}_j = [a_{xj}, a_{yj}]$ and $\mathbf{d}_j = [d_{xj}, d_{yj}]$, $1\le j\le D$, $2^5\le D<2^6$, which are known as the approximation and detailed coefficients. Only the approximation vector $\mathbf{a}_j$ will be used, in particular,

$$a_{xj} = \frac{x^*_{2j-1} + x^*_{2j}}{\sqrt{2}} \tag{4}$$

represents the X-graph and

$$a_{yj} = \frac{y^*_{2j-1} + y^*_{2j}}{\sqrt{2}} \tag{5}$$

represents the Y-graph. The new extracted feature $\mathbf{a}_j$ is then used for classification.

Let $\mathbf{b}_j = [b_{xj}, b_{yj}]$ represents the approximation vector of the corresponding character in the database obtained in the same manner as the vector $\mathbf{a}_j$. The notations in Equation 4 and Equation 5 follow [9].

Hence, the MULFR model is in the form of

$$\mathbf{a}_j = \mathbf{A}_j + \boldsymbol{\delta}_j \tag{6}$$

$$\mathbf{b}_j = \mathbf{B}_j + \boldsymbol{\varepsilon}_j \tag{7}$$

where $\mathbf{A}_j = \left[A_{xj}, A_{yj}\right]$ and $\mathbf{B}_j = \left[B_{xj}, B_{yj}\right]$ are two linearly related unobservable true values of $\mathbf{a}_j$ and $\mathbf{b}_j$ such that

$$\mathbf{B}_j = \boldsymbol{\alpha} + \beta\mathbf{A}_j$$

E. Some Results

Table 4 clearly shows that R_p^2 is better measure of similarity when compared to CBDD, MD, MQDF and CMF. This is especially true for the non-normalized characters. Further, the more experience writer is associated with higher R_p^2 values, suggesting that the numerical value of R_p^2 is a measure of writer ability.

V. A Compression Problem

For purposes of brevity, details of the compression problem are omitted but are available in [10]. One important result from [10], is that R_p^2 increases monotonically with respect to the compression factor as illustrated in Fig. 8. In particular a threshold R_p^2 value may be selected to correspond to a selected threshold compression factor.

Table 4: Experimental results for different writers: (a) with normalization and (b) without normalization. Each writer writes all 3000 different Chinese characters.

(a)

Distance measures		CBDD	MD	MQDF	CMF	R_P^2
Recognition rate (%) with normalization	Writer A	97	98	98	97	98
	Writer B	93	94	94	94	96

(b)

Distance measures		CBDD	MD	MQDF	CMF	R_P^2
Recognition rate (%) without normalization	Writer A	70	79.6	81.6	75	98
	Writer B	56	66.7	66.7	55	96

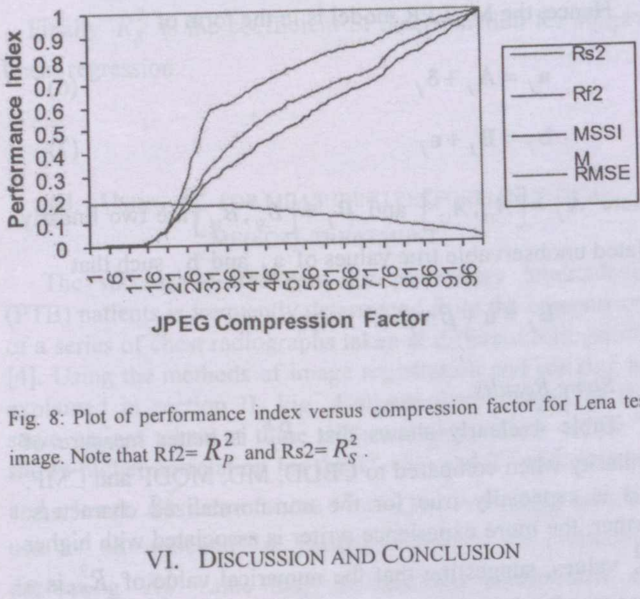


Fig. 8: Plot of performance index versus compression factor for Lena test image. Note that Rf2= R_P^2 and Rs2= R_S^2 .

VI. Discussion and Conclusion

Four different problems where R_P^2 has been used as a measure of performance strongly suggest that the correlation measure considered is potentially useful as a performance measure in a wide range of imaging problems. To generalize the result, more work has to be done, for example, repeating the character recognition problem with a larger database. Promising results were in fact obtained when the SCUT-COUCH2009 database, was studied in the character recognition problem [11]. A similar situation occurred when the compression problem was applied to the USC-SIPI Image Database [12] and LIVE database [13].

Properties of R_P^2 were also studied and shown to be unbiased and consistent estimators. A simulation study has shown that the performance of R_P^2 is robust to mild deviation from normality. Work in currently carried out to investigate the effects of dependent error terms.

In summary, R_P^2 shows the potential of being a measure of performance for many imaging application.

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