Efficiency Maximization of Resonant Coupled Wireless Power Transfer System via Impedance Matching Based on Coupling Tuning

Surajit Das Barman[1], Dr. Ahmed Wasif Reza[2] and Dr. Narendra Kumar[3]

Department of Electrical Engineering,
Faculty of Engineering, University of Malaya.
Kuala Lumpur, Malaysia-50603.
surajitbarman012@gmail.com, wasif@um.edu.my, narendra.k@um.edu.my

***Abstract*:** Non-radiative wireless power transfer (WPT) system using strong magnetic resonant coupling has been recently proposed and has shown more prospects in mid-range wireless power transmission due to its operating range and efficiency. The efficiency of such WPT system changes with the coupling distance between the coils changes, which is a critical issue for high efficient power transmission. In this paper, we demonstrate an impedance matching approach based on coupling tuning to enhance and maximize the efficiency of the system. The matching conditions are derived by analysing an equivalent circuit model of the system using reflected load theory (RLT) along with the derivation of maximum transmission efficiency. The matching condition is achieved by tuning coupling coefficients between the loop-coil and the internal coil, and verified against theoretical data by simulation in an electric automatic tool. Applying this technique results, efficiency of over 90% at the small coupling distance at a fixed operating frequency. The results show improvement of efficiency are about 16.1 % and 9.8 % at 90 cm and 1.15 m, respectively, compared to the traditional WPT system without matching, and thereby increase the operating range of the system.

***Keywords*:** wireless power transfer, magnetic resonant coupling, impedance matching, coupling, reflected load theory, transmission efficiency.

1. Introduction

Nowadays, wireless power transfer (WPT) technology is becoming a top research field for its effective and reliable applicability: from biomedical implants to charging portable electronic devices and high power electric vehicles (EVs), etc. Presently, electromagnetic induction coupling [1, 2] and microwave power transfer [3] are reported to be most popular WPT technology for consumer application. But the electromagnetic induction method is limited for short operating range, i.e., typically shorter than 30% of the coil dimension, and microwave power transfer suffers from low efficiency as it uses radiation. For mid-range operation the potential breakthrough of this wireless powering has occurred when the feasibility of the WPT system based on strong magnetic resonant coupling has been proven by MIT [4, 5] in both theoretically and experimentally.

Based on the principle of magnetic resonant coupling, two resonating coils tuned at the same frequency can effectively exchange energy with greater efficiency at a long operating distance compared to induction method. By using additional intermediate coils as a repeater the power transfer capability of the system can be extended. However, the power transfer capability of the system inversely varies with the distance or the axial misalignment between the coils. Therefore, maximizing efficiency and improving the charging capability over large distance has gained sizeable consideration to make the WPT system more practical. Recent resonant coupled approach [6] has proposed multi coil inductive link approach to increase the transmission efficiency of the WPT system at a large coupling distance by utilizing the coupled mode theory (CMT). But CMT only provides accurate results in case of having a weak coupled coil with a large quality factor which is not very convenient to explain the power transfer principle of the magnetic resonant system properly. Recently, a method of automatic adjusting of the source frequency has been suggested in [6] to deal with the distance variation and improve transfer efficiency; but this technique often tends the resonant frequency of the system to move out of the usable Industrial, Scientific and Medical (ISM) band of the mid-range WPT system. On the other hand, impedance matching technique using additional matching network on the transmission side reported in [7] satisfies ISM band; however, without considering the adaptive matching on the receiving side high transmission efficiency cannot be achieved and the additional matching networks increase losses of the system.

In this paper, we present a method of maximizing transmission efficiency of the WPT system by applying the principle of impedance matching at both the transmitting and receiving sides when the distance between the coils is changed. To set up the matching condition an equivalent series-compensated circuit model of the conceptual WPT system via resonant coupling is modelled and analysed by using reflected load theory (RLT) [8]. Empirical equations for the coupling parameters are established based on impedance matching principle, and the maximum efficiency of the system is calculated as a function of the loaded quality factors and coupling coefficient of the intermediate coil pair. This method neither require manual tuning of the source frequency nor additional lossy matching networks to increase efficiency, and is developed based on the tuning of coupling factors between the loop-coil and the internal coil of the designed system. Its feasibility is studied with the simulation at an electrical design tool.

2. Power Transfer Model of Resonant Based System

2.1 System overview

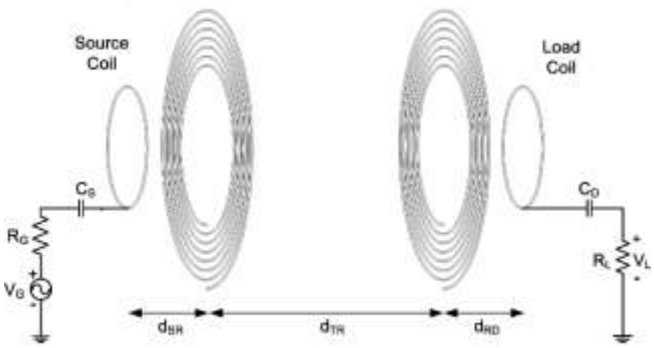


Figure 1. Schematic of the conceptual resonant coupled wireless power transfer system.

Figure 1 shows the schematic diagram of a magnetic resonant coupled wireless powering scheme consists of two internal multi-turn coils (TX and RX) and two loop coils (source and load coils). Both the loop coils have been modelled as a series-compensated resonant circuit with the external capacitor. This structure makes the compensation capacitance at both loop-resonators are independent of the load and maintains resonance even at varying load. The key interaction for energy transfer occurs mainly between TX and RX coils and the transfer efficiency of the system is practically depends on the distance d_{TR} between them. The other two distances between the coil and loop coil combinations are denoted by d_{ST} and d_{RD}, respectively.

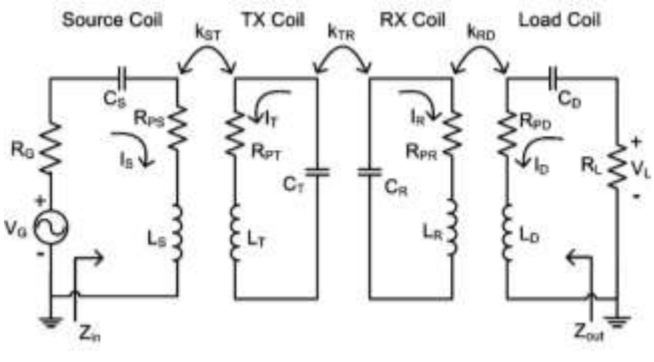


Figure 2. Equivalent circuit model of four coil wireless power transfer system.

The coils used for power transfer in the conceptual WPT scheme can be simply represented in terms of lumped circuit elements (L, C and R), as shown in Figure 2. The source coil is excited by a voltage source of amplitude V_G with an internal source impedance of R_G. The source and TX coils (load and RX coils) are inductively linked by coupling coefficient k_{ST} (k_{RD}) and the TX and RX coils are connected by coupling coefficient k_{TR}. These coupling factors can be calculated by using (1).

$$k_{AB} = \frac{M_{AB}}{\sqrt{L_A L_B}} \tag{1}$$

where M_{AB} represents the mutual inductance between the self-inductance L_A of coil A and L_B of coil B, respectively. To illustrate power transfer model of the proposed WPT system, we consider each coil model as a flat spiral resonator having a radius of r and a cross-sectional radius of a. Therefore, the self-inductance of each coil can be stated as

$$L_{self} \approx \mu_0 \, r \left[\ln\left(\frac{8r}{a}\right) - 1.75 \right] \tag{2}$$

When two coils having N_A and N_B turns, respectively, are aligned coaxially with a coupling distance of d_{AB}, the mutual inductance between the coils can be formulated in the Neumann form as

$$M_{AB} \cong \frac{\pi\mu N_A N_B r_A^2 r_B^2}{2\left[d_{AB}^2 + r_A^2\right]^{\frac{3}{2}}} \tag{3}$$

2.2 Transmission efficiency analysis

In Figure 2, we observe that all the coils are magnetically coupled to each other; therefore, the energy per cycle can be determined by the amount of the coupling. The analysis of the power transfer model of the system has been carried out by adopting circuit based reflected load theory. In this analysis, we neglect the cross-coupling terms (k_{SR}, k_{TD} and k_{SD}) which means neglecting reflected impedances onto the source coil, the TX coil and the source coil respectively from their corresponding link-up coils. To ensure effective power transfer with high quality factor of the coil we assume, $L_S \ll L_T$, $L_D \ll L_R$, $k_{TR} \ll k_{ST}$ and $k_{TR} \ll k_{RD}$. If $\omega_0 = 1/\sqrt{L_i C_i}$ and $Q_i = \omega_0 L_i / R_i$ represents the resonant frequency and the loaded Q-factor of ith coil, respectively, then at resonance we obtain following expressions:

$$j\omega_0 L_S + \frac{1}{j\omega_0 C_S} = j\omega_0 L_D + \frac{1}{j\omega_0 C_D} = 0\,;$$

$$j\omega_0 L_T + \frac{1}{j\omega_0 C_T} = j\omega_0 L_R + \frac{1}{j\omega_0 C_R} = 0\,;$$

$$Z_S = R_G + R_{PS} = R_S\,; \quad Z_T = R_{PT}\,;$$

$$Z_D = R_{PD} + R_L = R_D\,; \quad Z_R = R_{PR}\,. \tag{4}$$

Now, based on reflected load calculation and (8), we are able to find the individual efficiency of each coil of the system such that

$$\eta_D = \frac{R_L}{R_D}\,;$$

$$\eta_R = \frac{k_{RD}^2 Q_R Q_D}{1 + k_{RD}^2 Q_R Q_D};$$

$$\eta_T = \frac{k_{RD}^2 Q_R Q_D}{1 + k_{TR}^2 Q_T Q_R + k_{RD}^2 Q_R Q_D};$$

$$\eta_S = \frac{R_S \left[\dfrac{k_{ST}^2 Q_S Q_T (1 + k_{RD}^2 Q_R Q_D)}{1 + k_{TR}^2 Q_T Q_R + k_{RD}^2 Q_R Q_D} \right]}{R_{PS} + R_S \left[\dfrac{k_{ST}^2 Q_S Q_T \left(1 + k_{RD}^2 Q_R Q_D\right)}{1 + k_{TR}^2 Q_T Q_R + k_{RD}^2 Q_R Q_D} \right]}. \qquad (5)$$

Therefore, the magnitude of the forward transmission ratio (S_{21}) and the overall power transfer efficiency (η) of the system can be expressed respectively as

$$|S_{21}| = \frac{2 k_{ST} k_{TR} k_{RD} Q_T Q_R \sqrt{Q_S Q_D}}{1 + k_{ST}^2 Q_S Q_T + k_{TR}^2 Q_T Q_R + k_{RD}^2 Q_R Q_D + k_{ST}^2 k_{RD}^2 Q_S Q_T Q_R Q_D} \qquad (6)$$

$$\eta_S = \eta_S \eta_T \eta_R \eta_D \qquad (7)$$

3. Impedance Matching to Maximum Efficiency

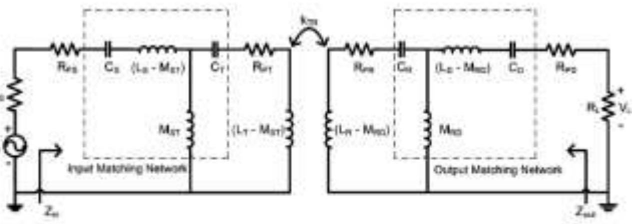


Figure 3. Equivalent circuit schematic of the conceptual impedance matching approach. (Here, $M_{ab} = k_{ab}\sqrt{L_a L_b}$ represents the extent of optimum magnetic flux linkage between corresponding coils to maximize efficiency).

Figure 3 illustrates a generic impedance matching system for conceptual resonant based wireless power transfer scheme to improve and maximize transfer efficiency. Comparing with Figure 2, we can observe that the conceptual equivalent impedance matching system can be constituted by C_S, L_S, C_T and the coupling k_{ST} at the transmitting side, and C_R, L_R , C_D and the coupling k_{RD} at the receiving side, respectively (Fig. 5). Therefore, by tuning the coupling coefficients between the loop-coil and the internal coil combinations, one can set the matching conditions in order to transfer preordained amount of power at the maximum efficiency. According to the principle of maximum power transfer, the input matching network acts to set the input impedance (Z_{in}) at the transmitting side equals to the complex conjugate of the optimal source impedance (R_G) of the power source which results the explicit equations for tuning the coupling coefficient k_{ST} and the input impedance Z_{in} given by

$$k_{ST} = \sqrt{\frac{1}{Q_S Q_T} \left(\frac{R_G - R_{PS}}{R_S} \right) \sqrt{1 + k_{TR}^2 Q_T Q_R}} \qquad (8)$$

$$Z_{in} = R_{PS} + R_S \left[\frac{k_{ST}^2 Q_S Q_T \left(1 + k_{RD}^2 Q_R Q_D\right)}{1 + k_{TR}^2 Q_T Q_R + k_{RD}^2 Q_R Q_D} \right] \qquad (9)$$

Similarly, the output matching network also acts to match the output impedance (Z_{out}) at the receiving side with the conjugate of the optimal load impedance (R_L) for achieving peak transfer efficiency. This matching condition acquires the equation for the coupling coefficient k_{RD} and the optimum load derived as

$$k_{RD} = \sqrt{\frac{1}{Q_R Q_D} \sqrt{1 + k_{TR}^2 Q_T Q_R}} \qquad (10)$$

$$R_{L(opt.))} = R_{PD} \left[\frac{k_{RD}^2 Q_R Q_D}{\sqrt{1 + k_{TR}^2 Q_T Q_R}} - 1 \right] \qquad (11)$$

Based on (7)-(11), the calculated maximum transfer efficiency can be expressed as

$$\eta_{\max} = \frac{k_{TR}^2 Q_T Q_R}{\left[1 + \sqrt{1 + k_{TR}^2 Q_T Q_R}\right]^2} \cong |S_{21}|^2 \qquad (12)$$

From (12), it is shown that the theoretical maximum power transfer efficiency is approximately equivalent to the square of the linear magnitude of S_{21}, i.e., the conceptual WPT system based on resonant coupling providing maximum power to that load acts as a two port network.

4. Validation

To verify the proposed matching approach based on coupling tuning, we set up the equivalent circuit model of Figure 2 in Advance Design System (ADS) simulation software. The S-parameters toolbox is used to measure the input and output impedances and adjust the desire couplings, respectively, for achieving maximum efficiency. The power probe toolbox is used to calculate the maximum transmission efficiency and compare the extracted results with the theoretical value calculated by the derived equations discussed earlier.

4.1 Coil parameters

For simplification, the TX and RX coils of the designed WPT system are considered as identical flat spiral coil with approximately 8 turns, having pitch of 1 cm and outer diameter of 29 cm. On the other hand, both the loop-coils (source and load coils) is 20 cm in diameter having single turn. The cross-sectional radius of the coil wire is assumed to be 2 mm. The extracted lumped parameters for each coil are given in Table I. The resonant frequency of 12 MHz is set for both TX and RX coils, but for loop-coils, additional capacitors are connected in series to tune them at 12 MHz.

Table 1. Extracted electrical parameters of each individual coil.

Coil	Inductance (μH)	Self-resistance (Ω)	Capacitance (pF)
Source	0.474	0.213	371.11
TX	22.22	4.98	7.93
RX	22.34	6.36	7.87
Load	0.481	0.2186	365.7

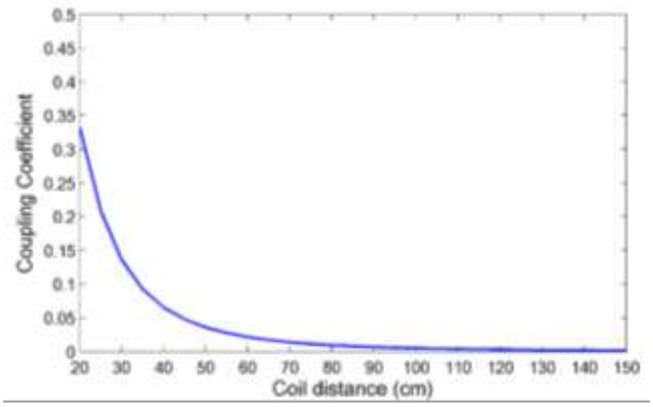
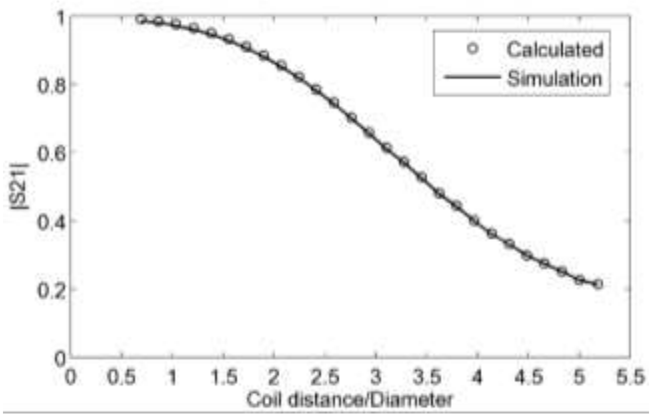


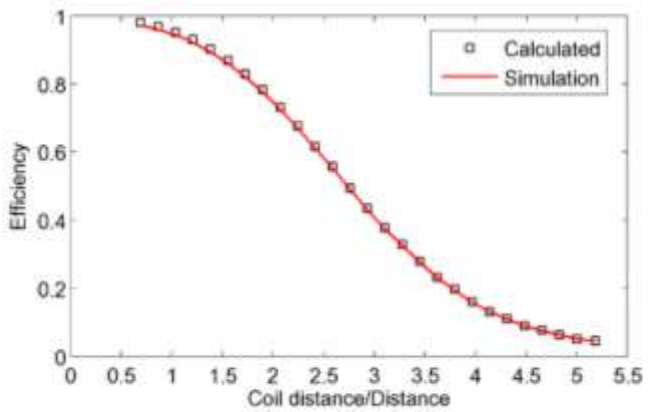
Figure 4. Coupling coefficients vs. coil distance.

4.2 Circuit simulations

The proposed WPT circuit model with a conceptual matching approach have been designed using the symbolic defined device (SDD) in the ADS software that includes (4)-(12). The mutual inductance and couplings between two coils is extracted using (3) and (1), respectively. A sinusoidal power source having an internal resistance of 50 Ω is used to supply power to a load of 50 Ω resistance. The Q-factor of each loop coil becomes limited at the resonant frequency of 12 MHz because of the high series resistance connected. Figure 4 illustrates the extracted value of coupling coefficient k_{TR} for a distance range of 20-150 cm between TX and RX coils. The input power supplied to the system will change with the coil distance as the load impedance seen by the source is a strong function of distance variation. Initially, the distance and are assumed to be fixed in such that the coupling value k_{ST} and k_{RD} become equal to 0.2527 and 0.2486 respectively. The coupling coefficient decreases with the increase of distance and vice-versa. Therefore, to establish the conceptual impedance matching method, we optimally tuned the couplings and with respect to the distance using (8) and (10), respectively.



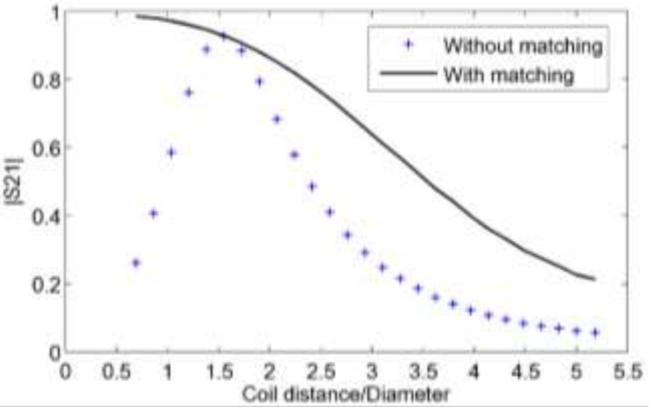
(a)



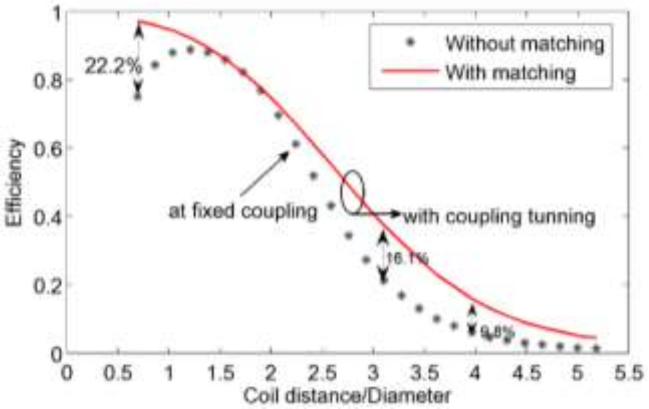
(b)

Figure 5. (a) Calculated and simulated $|S_{21}|$ vs. normalized coupling distance to the resonator diameter, (b) measured and simulated transmission efficiency vs. normalized coupling distance to the resonator diameter of 4-coil WPT system at a single resonant frequency with proposed impedance matching based on coupling tuning.

We implement the coupling tuning technique in ADS to develop the impedance matching conditions and extracted the forward transmission coefficients S_{21} and transmission efficiency is measured. The analytical comparison of the theoretical and simulated values of S_{21} and efficiency at the resonant frequency (12 MHz) versus the normalized coupling distance (d_{TR}) between TX and RX coils to the coil diameter are illustrated in Figure 5(a) and Figure 5(b), respectively. As it is seen, the derived equation for maximum efficiency using impedance matching based on coupling tuning is almost generating similar results as the simulation except some slight deviation at the close distance. This difference is due to ignoring the parasitic cross-coupling parameters. This is due to ignoring the parasitic cross-coupling parameters (k_{SR}, k_{TD} and k_{SD}) in the theoretical analysis. But when the coils are near to each other, these parameters cannot be neglected. Therefore, for accurate analysis the effect of cross-coupling coefficients are taken in case of simulation. As observed, the efficiency reaches more than 90% at close proximity without changing the source frequency.



(a)



(b)

Figure 6. (a) Simulated $|S_{21}|$, and (b) efficiency comparison of the wireless power transfer system with and without impedance matching.

Using impedance matching technique via coupling tuning shows great improvements in both $|S_{21}|$ and efficiency comparing over the results obtained at fixed coupling without matching, which are illustrated in Figure 6(a) and 6(b), respectively. Without matching, both $|S_{21}|$ and efficiency becomes low at a close coupling distance because of the frequency splitting phenomenon which occurs due to the variation of impedance when the system operates in the strong coupling regime. The magnitude of the power transferred to the load is then getting higher with the distance and reached at its peak when the coils are critically coupled to each other. When the gap between the coils is increased further, their coupling becomes weak and makes the efficiency to fall down rapidly. In contrast, adjusting coupling coefficients k_{ST} and k_{RD} in case of matching the impedance during distance variation provides better results at fixed operating frequency and the maximum efficiency reaches more than 95% at a distance of 20 cm. The transfer efficiency improvements are about 16.1 % and 9.8 % at 90 cm and 1.15 m respectively. In experimental validation the efficiency may drop about 3% - 5% due to the radiation and ohmic losses of the resonators and components used in the system.

4.3 Compensation for load variation

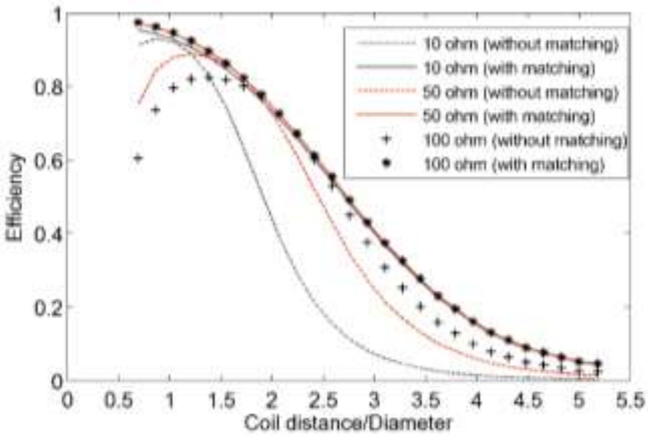


Figure 7. Efficiency comparison at different load resistance for the WPT system with and without impedance matching.

The conceptual impedance matching based on coupling tuning can successfully compensate the effects of load resistance variation compared to the traditional WPT system without matching depicted in Figure 7. Higher efficiency can be achieved at the close distance without matching for a low value of the load resistance as observed. For higher load resistance, a wide operating with good efficiency is possible without matching, but the small Q -factor of the load coil actually shifts and lowers the peak efficiency point compared to small load resistance. Contrarily, the proposed impedance matching technique maximizes and provides almost same efficiency curve at the original resonant frequency for different load resistances without changing the source voltage. The results also confirm the improvement of the operating range of the system with a maximum output power over the system without matching.

5. Conclusion

In this paper, an impedance matching approach based on coupling tuning has been demonstrated that can provide a precise characterization of resonant coupled wireless powering system and maximize power transmission efficiency of the system at a fixed operating frequency. Design guidelines for the system with a conceptual matching approach has analysed based on the equivalent circuit model, source and load impedances, and coupling coils. Empirical equations of maximum efficiency and coupling parameters have been developed based on applying impedance matching principle for optimal values of source and load coils. The effects of the variation of the coupling distance between coils has been analysed and the simulation results are in positive consent with the theoretical model. For small coupling distance, the proposed technique successfully compensates the shifting of resonant frequency and maximizes the efficiency up to 97%. Moreover, the proposed matching technique can reimburse the effect of load variation of the system and can transfer approximately the same amount of power to different loads at a fixed distance.

Acknowledgment

This research work is supported by the University of Malaya High Impact Research (HIR) Grant (UM.C/628/HIR/ENG/51) sponsored by the Ministry of Higher Education (MOHE), Malaysia.

References

[1] J. J. Casanova, L. Zhen Ning, and L. Jenshan, "A Loosely Coupled Planar Wireless Power System for Multiple Receivers," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 8, pp. 3060-3068, 2009.

[2] U. K. Madawala and D. J. Thrimawithana, "A Bidirectional Inductive Power Interface for Electric Vehicles in V2G Systems," *IEEE Transactions on Industrial Electronics,* vol. 58, no. 10, pp. 4789-4796, 2011.

[3] W. C. Brown, "The History of Power Transmission by Radio Waves," *IEEE Transactions on Microwave Theory and Techniques,* vol. 32, no. 9, pp. 1230-1242, 1984.

[4] A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljačić, "Wireless power transfer via strongly coupled magnetic resonances," *science,* vol. 317, pp. 83-86, 2007.

[5] A. Karalis, J. D. Joannopoulos, and M. Soljačić, "Efficient wireless non-radiative mid-range energy transfer," *Annals of Physics,* vol. 323, pp. 34-48, 2008.

[6] A. P. Sample, D. A. Meyer, and J. R. Smith, "Analysis, experimental results, and range adaptation of magnetically coupled resonators for wireless power transfer," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 2, pp. 544-554, 2011.

[7] B. Teck Chuan, M. Kato, T. Imura, O. Sehoon, and Y. Hori, "Automated Impedance Matching System for Robust Wireless Power Transfer via Magnetic Resonance Coupling," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 9, pp. 3689-3698, 2013.

[8] M. Kiani and M. Ghovanloo, "The circuit theory behind coupled-mode magnetic resonance-based wireless power transmission," *IEEE Transactions on Circuits and Systems I: Regular Papers,,* vol. 59, no. 9, pp. 2065-2074, 2012.