Assessing Algebraic Knowledge : Procedural or Understanding

By:

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ASSESSING ALGEBRAIC KNOWLEDGE: PROCEDURAL OR UNDERSTANDING

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Introduction

Calculus has traditionally one of the most difficult concepts for students to understand and master. Although most students learnt the specific algorithm and procedure that they are taught, their general conceptual understanding often remains remarkably deficient. Recent years have seen increasing interest in developing the use of writing in the mathematics classroom (Allen, 1992; Burton & Morgan, 2000 ; Borasi & Rose, 1989; Oaks & Rose, 1992; LeGere, 1991). This is largely based on the promise that writing is an activity that is in itself conducive to learning. The rules and procedures of school mathematics make little sense to many students. They memorise examples, they follow instructions, they do their homework, and they take tests, but they cannot say what their answers mean.

Most of us want our students to feel that mathematics is an enjoyable and rewarding study for them but do we succeed? Some of our students are successful whereas others are anxious and fearful. The achievement of mathematics has been generally been poor. Several students are not able to explain what they have done in solving mathematics and how they obtained the answers. The rules and procedures of school mathematics make little sense to many students. The question now is,"How do we know if we're teaching for understanding?"

Understanding and knowing mathematics is doing mathematics (Countryman, 1992, Dougherty, 1996). We need to create situations where students can be active, creative, and responsive to the physical world. Researcher believes that to learn mathematics students must construct it for themselves. They can only do that by exploring, justifying, representing, discussing, using, describing, investigating, and predicting. Writing is an ideal activity for such processes. Writing can motivate and enhance the learning that takes place when students confront the concepts and procedures of mathematics.

Writing is an important learning tool in mathematics education that encourage and enable students to value mathematics, gain confidence in their own mathematical ability, become problem solvers, communicate mathematically, and reason mathematically (NCTM 1989, 123). Vygostsky (1987) believes that writing requires the writer to compact inner speech maximally so that it is fully understandable, thus necessitating the deliberate structuring of a wed of meaning.

In an investigation of the benefit of using expository writing, Bell and Bell (1985) concluded that "expository writing is an effective and practical tool for teaching math problem solving" (p. 214). Writing helps build thinking skills for mathematics students as they become accustomed to reflecting and synthesizing as parts of a normal sequence involved in communicating about mathematics (Pugalee, 1997). Writing should be encouraged as an integral part of the mathematics curriculum designed to help students in understanding mathematical concepts.

Conceptual Framework

An important goal of mathematics instruction is that students should understand. But, what exactly do we mean by understanding mathematics? Wong (1984) states there is much ambiguity and lack of well defined criteria over the meaning of mathematical understanding. Writing in mathematical understanding have been based mainly on rational analysis and expert opinion.

Likewise for mathematics, which we may indeed be trying to "sell" more actively than some of our pupils are wanting to "buy". Although the subject matter (mathematics) may be the same, for various persons and on various occasions the goals of learning may well be different, with the likelihood that different kinds of schema may be appropriate. So if we are trying to answer the question, "How many kinds of understanding can we usefully distinguish?" where "we" means ourselves as mathematical educators, and usefully now takes the meaning , "appropriate to the different kinds of leaning goals set by our pupils and students." The distinction between various kinds of leaning goal.

We must also remind ourselves that the goals of teachers and pupils may differ. Moreover if we set up a learning goal, this will influence the kind of learning goal, this will influence the of schemas pupils construct at least as much as, possibly more than, what is overtly being taught. So there will be mismatch between the situationally-determined learning goals, and those which teachers may have in mind and/or verbally present. According to Skemp (1979) there were two kinds of assessing.

Procedural

Instrumental Understanding is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works. In other words "how" but does not know "why" (Skemp, 1979). Instrumental Understanding applied to the concept of average consist of knowing only the computational rule for the calculating of simple average of a set of numbers. In this study the terms instrumental understanding, computational knowledge, computational ability, computational skill, procedural skill and procedural knowledge were used interchangeably when referring to instrumental understanding.

The schemas formed by instrumental learning are short-term, the least and most quickly acquirable by which correct answers can be given; in other words rules, which we may regard as degenerate schemas. Pupils learn a set of these, each appropriate to a limited class of task. These rules are not entirely isolated—they can be combined, and applied in succession. But the mental structures acquired by instrumental learning have limited adaptability, because these rules are ways of manipulating symbols, and the connections are between symbols and not concepts. Application to substantially new situations requires conceptual connection, which is to say relational schemas.

Understanding

It has been suggested that this category is unnecessary, being no different from relational understanding. But suppose, that a pupil writes something like

$$
\begin{aligned}
Y + 3 &= 7 \\
&= 7 - 3 \\
&= 4
\end{aligned}
$$

We ask why he writes this, and he answers in a way which convinces us that he has relational understanding. (Eg. "The top line says that adding 3 to y gives us 7, so to find whatyx is we take the 3 away again.")

If we now point to him the exact meaning of what he has written, and he then re-writes it correctly, this would be evidence that he was capable of logical understanding, which we had helped to bring about by causing him to reflect. By ".... Re-writes it correctly" we mean in an acceptable form which sets down a valid sequence of logical inferences.

Logical understanding is closely related to the difference between being convinced oneself, for which relational understanding is sufficient, and being able to convince other people.

Most of us will also subject our ideas to self criticism before making them public, formally or informally and the construction of proof which satisfies ourselves gives us confidence to do so by writing out the explanation. In some cases, all we want is to satisfy ourselves. But I would regard the satisfaction of criticisms and self-criticism as secondary to the main goal, and indeed as aids to its achievement, the construction of ever more extensive and powerful mathematical knowledge, coherent, and without weaknesses or internal inconsistencies. This is clearly quite an activity; writing proofs of this sort, of a formal nature, were learnt by reflecting Thus the most sophisticated level, valuable only to those who have achieved real insight and then recognized the need for proof is linked with the most primitive, thus firmly ensuing that the matter will not be understood!

Communication through Writing

The learning of mathematics is a complex and dynamic process involving interactions between previously acquired levels of understanding and the conceptualization and incorporation of new material. Writing encourages a level of cognitive activities which maximizes the potential of the learner to modify and restructure mathematical knowledge.

As suggested by National Council of Teachers of Mathematics (1989) the ability to communicate mathematics involves being able to:

1). express mathematical ideas by speaking, writing, demonstrating, and depicting them visually;

2). understand, interpret, and evaluate mathematical ideas that are presented in written, oral, or visual forms;

3). use mathematical vocabulary, notation and structure to present ideas, describe relationships, and model situations (p. 214)

If we believe that students learn best by constructing and evaluating the knowledge, that we wish them to acquire, we are likely to view students as research apprentices who gain ownership of knowledge by raising their own questions about existing knowledge.

Writing definitely the key role in the process of student knowledge-construction (Guckin, 1992; Luitel, 2002; Mayer & Hillman, 1996) . Writing can help teachers answer specific questions about students;

1). Do students use math to make sense of complex situations?
2). Can they formulate hypotheses?
3). Can they organize information?
4). Are they able to explain concepts?
5). Can they use computation skills in context?
6). Do they use mathematical language appropriately?
7). Are they confident about using mathematical procedures?

Objectives of the Study

This purpose of this study was to explore and investigate the use of writing activities as a tool in enhancing teaching and active learning of selected topics in Malaysian secondary school mathematics.

Specifically this research project seeks to:

1. find out the effects of writing activities on student achievement in calculus;
2. find out the effects of writing activities on student attitude toward learning calculus and toward the subject;

Specifically, it will seek to find answers to the following questions:

1. Does writing activities help improve student achievement in calculus?
2. Does writing help to improve the attitude of students toward learning concepts in calculus?
3. What are the perceptions of students towards writing activities in calculus?

Significance of the Research Study

This study proposed a significant idea on the use of writing activities as a tool in enhancing teaching and learning secondary mathematics. Findings of this study shall:

1. provide information to schools to take advantage of the writing activities and mathematical versatility;

2. provide additional body of knowledge or literature on the use of the writing activities in the Malaysian context;

3. becomes the basis for producing instructional materials suitable to be use in the secondary mathematics curriculum.

Methodology

Sample. The research study employed the quasi-experimental, non-equivalent control pretest and posttest design. The researcher in this research conducted a series of training workshops with teachers on how to use the writing activities as a tool in mathematics. The sample of the study consisted of two experimental secondary schools in Selangor. Each school was assigned one intact class of Form Four to be the experimental group and another one intact class to be the control group. The experimental group learned mathematics by using the writing activities for five weeks, while the control group learned mathematics by using traditional whole-class instruction. The mathematics teachers who teach in the experimental group were trained in the workshops.

Instruments. The instruments in this study consisted of:

1). Instructional Materials -Module/worksheets to be used in the training workshop and in classroom for five weeks;
2). Calculus achievement test,
3). Attitude Inventory Items; and
4). Questionnaire for students' perceptions.

Validity and Reliability. A 20-item Calculus Achievement Test was designed for this study. A test-retest reliability in this pilot test is .87. After the test had been prepared, three experienced mathematics teachers with more than fifteen years of teaching experience were requested to check the questions in this test for content validity. Attitude Inventory items and questionnaire for students' perceptions has been validated by the same teachers.

Procedure. Both quantitative and qualitative methods were utilized to gather data. Quantitative data was collected using a pretest and posttest Calculus Achievement Test, Attitude Inventory Items, and a questionnaire for students' perception. Qualitative data on the other hand was collected by means of interview with twelve students. The interview sessions were audiotaped.

Different instructional materials were used for the experimental and control groups. The researcher prepared 16 expository writing worksheets for the experimental group whilst the control group used the textbook. In every teaching session, researcher used writing activities for experimental group. The students need to do the following activities:

1). Describe each step used in solving the problem clearly
2). Explain why those steps were used
3). How the students used the previous knowledge in helping them to solve the problem
4). Describe in detail what is(are) the thing(s) the student think while they solve the problem.

Examples of the questions are as follows:

1. Differentiate $5x^5 - 12x^4$ with respect to x. Explain how do you differentiate the problem in detail.
2. Given $y = (4x^3 + 6)/x$. Describe each steps you use to differentiate the given problem and give reason for your choice of steps.
3. In words, write what you would do to solve this problem. Where appropriate, tell why you are doing that step to the problem $5x^3 + 40x^2 = -30x$.

Data Analysis. Data from students' achievement in pretest and posttest, attitude and questionnaire were analyzed using quantitative analysis. The SPSS program was used to analyze the data. The quantitative analyses were complemented with analyses of qualitative data gathered as a result of interviews.

Results and Discussion

Student Achievement in Mathematics

To answer the question whether students in the experimental group using writing activities achieve significantly greater improvement on mathematic scores compared to students in the control group who do not use the instructional activities an analysis of covariance (ANCOVA) was used.

The ANCOVA shows a main effect for usage of writing activities, $F(1,167) = 56.38$, $p<.05$, indicating significant difference on the improvement scores in mathematics between the experimental and the control groups.

To further examine the data for differences between the two groups, the adjusted mean scores of achievement posttest of the two groups were determined. Table 1 provides a summary of the adjusted means of the experimental and control groups of subjects.

Table 1

Means and Standard Deviations for Experimental and Control Groups on Pre- and Posttest Mathematics

Test		Experimental	Control
Covariate (Pretest)	N	85	84
	Mean	12.01	12.29
	Standard Deviation	3. 91	3.88
Dependent (Posttest)	N	85	84
	Mean	24.17	20.21
	Standard Deviation	3. 12	3.24
	Adjusted Means	24.51	20.52

The pretest mean for the experimental group was 12.01 (SD=3.91) compared to the control group mean of 12.29 (SD= 3.88). The posttest means for both groups increased from the pretest, with experimental group showing the greater increase. Table 1 shows that the adjusted mean of the experimental group was significantly higher than the adjusted mean of the control group.

The results of the analysis of the usage of writing activities indicated that students in the experimental group showed significantly greater improvement on mathematics achievement than students in the control group.

Attitude of students toward learning in Calculus

The following Table 2 summarizes results from 85 Form Four students. All survey questions were phrased so that an "Agree" or "Yes" answer is a favorable response and "Disagree" or "No" answer is an unfavorable response.

Table 2

Attitude of Students toward Learning in Calculus

Item	Favorable	Unfavorable

	(%)	(%)
1. I like Calculus better now	79	21
2. I learned more Calculus now	84	16
3. I spent more time on Calculus now than before	85	15
4. I enjoy Calculus better now than before	87	13
5. It was easy to learn Calculus by *writing activities*	83	17
6. I learn Calculus better by reflecting instead of only with book and memorize	83	17

The students showed positive reaction towards the use of writing activities. This is reflected in the responses they gave in the survey form. Table 2 showed 79% of the students like Calculus better, 84% learned more Calculus, 85% spent more time on Calculus and 87% enjoy Ccalculus better now than before. Students also highly agreed that learning Calculus was easy and they also agreed (83%) that they learn better by reflecting through writing instead of only using a book and memorize.

Perceptions of Students towards the Use of Writing Activities

Eighty five form four students completed a form with the questions as shown in Table 3. The scaled score is calculated based on 5 – strongly agree, 4 – agree, 3 – not sure, 2 – disagree, and 1 – strongly disagree.

Table 3
Students' Perception towards the Use of Writing Activities

Item	5	4	3	2	1	Min
1. Writing activities help me in understanding the topics better.	7 (8.24)	68 (80.00)	5 (5.89)	5 (5.89)	0 (0.0)	3.91
2. I am able to interact with my teacher & friends	12 (140.12)	59 (69.41)	10 (11.76)	3 (3.53)	1 (1.18)	3.92
3. I feel confident about trying a new problem.	11 (12.94)	66 (77.64)	6 (7.06)	2 (2.35)	0 (0.00)	4.01
4. Writing activities make me feel comfortable learning Calculus	9 (10.59)	67 (78.82)	5 (5.89)	3 (1.18)	1 (1.18)	3.94

As shown in Table 3, most of the students showed positive reactions towards the use of writing activities. Students felt confident about trying a new problem with a min 4.01. Students felt that writing activities made them comfortable learning Calculus, min 3.94.

Belief and Attitudes about Writing Activities as Learning Tool

As the interview data were analyzed, it became clear that the students beliefs and attitudes about the use of writing activities in mathematics learning actually fell into several categories:

1). Concepts involved encourages students to focus on their own thinking and use their own language.

The researcher found that the students' explanation showed that students were investing in a mathematical writing task is dependent not only on the value they recognize in the task but also on their perception and reflection of their own thinking and the use of their own language. In the experimental group, the following was found:

> *The students were able to explore algebraic functions and describing how to integrate the function using their own words. By doing so, the students had logically deduced the meaning of the gradient.*

Those who were not confident in their mathematical abilities tend to stop working and explaining on the task as they become perplexed while the more confident students tolerated perplexity longer and were likely to continue with the task.

2). Conceptions and misconceptions will be revealed as students describe their explorations of a problem.

3). Writing activities assist students to explore multiple methods and multiple solutions.

4. Students appreciate the opportunity to become authors of their own ideas.

5). Writing about their work gives students a chance to experience a creative side of Calculus.

6). Writing encourages students to explore content rather than merely concentrate on the mechanics of symbol manipulation.

Conclusion

Students who have succeeded on a task are usually eager to do more of the same kind of task. They are motivated to achieve learning goals that they consider relevant to their needs. A learning goal is an instructional purpose, aim or objective that is set before students as a means of encouraging learning (McIntosh & Draper, 2001). Students usually aim to achieve goals that they perceive as interesting, realistic and attainable. Mathematics can be a creative activity involving intuition and invention (Miller, 1992). Mathematicians often explore mathematical ideas with a specific goal and discover new and interesting relationships through writing.

In this research, students were given an opportunity to use writing activities to explore mathematical materials, concepts and ideas freely to assist them to develop their own intuitive ideas about mathematics.

As we know that mathematics students often need time to think about the problem before gaining an insight into possible solutions (Pugalee, 1997). Therefore in this research, students in the experimental group were given similar opportunities. Students' insights may open up further possibilities for creative endeavor. The writing which involved reflecting problem-solving process is an exciting and creative process for students and teachers. When solving problems students involved in creative processes such as:

1). Creating and recognizing patterns

2). Searching for alternative methods of solving a problem

3). Experimenting with different ways of communicating mathematical ideas

4). Creating personal hypotheses

When students perceive learning to be interesting, fun, personally meaningful, and relevant and the context supports and encourages personal control, motivation to learn and self-regulation of the learning process occur naturally (Brophy, 1987; Lepper, 1988; Noraini, 1999). Learning activities and experiences that students find interesting and stimulating are usually inherently motivating. When students' interests in prescribed learning have been aroused, there is usually little need for other incentives or reinforces. To make learning interesting and challenging to students, there needs to be sufficient variety in the nature and type of planned activities. In this research, the researcher and teachers involved the teaching of experimental class selected learning tasks that are challenging and varied, so that students will be intrinsically motivated to complete the tasks.

To take advantage of the vast potential of writing activities for all students, teachers will need to make significant changes in their pedagogy. They will need to form working relationships with students that allow for creative, higher order thinking to emerge. They will need to break traditional barriers that impose restrictions on the potential benefits to be realized from technology in classrooms, and they will need to be willing to make ongoing efforts to be successful in the implementation process.

Changing pedagogy expects that teachers will be proficient at using writing as an instructional tool. In addition, teachers may no longer follow the well-traveled road of traditional education, but must be visionary in the potential opportunities for learning and their shift from teacher to guide and facilitator. The teachers need to structure mathematics lessons differently. They have to change in terms of the entire process, focus, and outcomes of educational expectations. Teachers are not only transferring knowledge to students but they are also engaging in different roles as students experience different processes of learning.

The writing activities were able to motivate the learner, identifying what is to learned, and providing active involvement. With the use of writing activities, students were also able to compare, classify, analyze errors, or construct support that they encounter in the course of solving problem.

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Please write all the information requested in the table below. Please send this completed form to Professor Shuhua An at san@csulb.edu, and Professor Yiming Cao, at caoym@bnu.edu.cn **by Tuesday, April 28, 2009.**

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Title of Presentation	Assessing Algebraic Knowledge: Procedural or Understanding?
Description of Presentation (about 200 words)	Students learn new algebraic concepts and procedures by building on what they already know. In other words, learning with understanding can be viewed as making connections between existing knowledge and new information. In many mathematics classroom today, children learn mathematics as acquiring the mastery of a set of predetermined procedures and skills. Teachers perceived their job as transmitting the content of mathematics by demonstrating the correct procedure and ensuring the children practice the skill of using these procedures. When children were able to provide the correct answer to the question posed by teachers, both teacher and children felt satisfied and thought that they have acquired the kind of thinking valued by society. In mathematics classroom, communication can be defined as sharing of algebraic understanding. In fact, Noraini Idris (2006) expressed that mathematics language is viewed as a medium through which mathematics content is communicated. Teachers should assess how well children have understood the algebraic knowledge through children performance in the classroom. In this presentation, the presenter will share the data in the Malaysian classroom on how the teacher assess his/her children's algebraic knowledge. Does teacher label the children as procedural or understanding? Presenter will

	also share how teacher assess children's understanding and their attainment of algebraic concepts. Children need to be given feedback on their progress. Will teacher be able to this through the process of teaching and learning? Presenter will also share on this issue.
Grade Band(s): K-5, 6-8, 9-12, College	9 - 12