CRACK DETECTION IN A SIMPLY SUPPORTED RC BEAM USING SIMPLIFIED LAPLACIAN

H. Abdul Razak [1], S.L. Teng [1], Z. Ismail [1], A. G. Abdul Rahman [2]

[1] Department of Civil Engineering,
[2] Department of Mechanical Engineering,
Faculty of Engineering,
University of Malaya,
Lembah Pantai, 50603 Kuala Lumpur,
Malaysia.

ABSTRACT

This paper describes the detection and identification of cracking severity in a reinforced concrete beam using modal data. Experimental modal test was performed on the beam before and after application of load of increasing intensity to induce a single crack at a fixed location. Modal parameters extracted were categorised into two groups, namely local response parameter (mode shape data) and global response parameters. Localised changes such as cracking reduce the stiffness of the structure, and therefore caused a localised change in the discrete function of the mode shape. From a previous research [1], Laplacian Operator was initially proposed. However, when applied to a non-homogenous and composite material such as reinforced concrete, the operator was too sensitive and did not yield meaningful results. Hence, a simplified Laplacian operator was suggested. In this investigation, it was found that global response such as natural frequency is a good indicator to detect the occurrence of the deterioration in the structure. However, the local response data such as mode shape was required to detect the defect location accurately and a simplified Laplacian operator worked quite successfully. Furthermore, it was observed that the higher frequency modes tend to detect the occurrence of cracking in the earlier stages.

NOMENCLATURE

n	number of primary/master measured DOFs
x,y,z	translational degrees of freedom/coordinates
ω_r/f_r	natural frequency of r^{th} mode
i	mode shape number
E	modulus of elasticity
I	second moment of a cross-sectional area
ζ	damping
SLO/LO	Simplified Laplacian Operator/Laplacian Operator

1. INTRODUCTION

The application of modal testing to determine crack development, propagation and detection has been extensively investigated over the past few decades. The effect of cracking on the dynamic behaviour has received much attention because of its importance in mechanical and civil engineering systems. When a structure suffers localised deterioration, such as cracking, its dynamic properties will change. Localised damage normally can cause a reduction in structural stiffness, which can lead to a decrease in natural frequencies and finally, changes in both modal damping and mode shapes. Among these modal parameters, natural frequency was used widely by most of the researchers as damage indicator, since it always returned satisfactory results in damage detection processes. However, lately mode shape data is getting much more attention from researchers. It was found that it could provide information about the occurrence of the damage and also to detect the location of the damage in the structure.

Chondros and *Dimarogonas* [2] studied the effect of the crack depth on the dynamic behaviour of a cantilever beam. They showed that increasing the crack depth reduces the natural frequencies of the beam. *Masoud*, *Jarrad* and *Al-Maamory* [3] studied the effect of crack depth on the transverse vibrational characteristics of a prestressed fixed-fixed beam. They found that the existence of a crack in a beam increase the local flexibility of the beam. The increase of the local flexibility is equivalent to the lowering of the local bending stiffness of the beam, and therefore, a decrease in natural frequencies was detected. *F. Ismail*, *A. Ibrahim* and *H. R. Martin* [4] investigated the effects of crack closure on the frequency changes of cracked cantilever beam under dynamic loading. Both computer simulation and experimental modal analysis were conducted. They concluded that relying on the drop in the natural frequencies alone, especially the higher mode might lead to serious underestimation of the crack severity. On the other hand, these results proved the reliability of vibration testing in detecting the presence and nature of the crack. *W. M. Ostachowicz* and *M. Krawczuk* [5] investigated the effects of two types of crack upon the natural frequencies of a cantilever beam namely double sided and single sided cracks. Results and conclusions similar to Ostachowicz were found. *A. K. Pandey*, *M. Biswas* and *M. M. Samman* [6] proposed a new parameter called curvature mode shape as a possible candidate for identifying and locating damage in a structure. By using a cantilever and a simply supported analytical beam model, it was shown that the absolute changes in the curvature mode shapes are localised in the region of damage and hence can be used to locate the damage in a structure. They also found that the changes in curvature mode shapes increase with increasing size of damage. Later, *C. P. Ratcliffe* [1] carried out the same procedure and applied a finite difference approximation of Laplace's differential operator to the mode shape. This operator, known as Laplacian operator successfully identified the location of the damage with a high degree of severity. However, for lower degree of damage, a modified Laplacian Operator was proposed.

Both FE simulation and experimental results were used to verify the technique.

The primary objective of this investigation is to establish a technique for assessing defects specifically load induced cracking in reinforced concrete beams, by examining the modal parameters. Modal tests on the beams are conducted to determine the modal parameters. By comparing data obtained, the effect of the damage on the modal parameters such as natural frequencies and mode shape can be studied. A simplified algorithm to operate on the mode shape data is proposed to ascertain the defect location in the structure.

2. ANALYTICAL TECHNIQUE

Curvature mode shapes are related to the flexural stiffness of the cross-section of the beam. Curvature at a point is given by

$$\upsilon'' = M/(EI) \qquad (1)$$

in which, υ'' is the curvature of the section. M is the bending moment at the section.

If a crack or other damage is introduced in the structure, it reduces the bending stiffness (*EI*) value of the structure at the cracked section or in the damaged region, which increases the magnitude of curvature at that section of the structure. The changes in curvature are local in nature and hence can be used to detect and locate a crack or damage in the structure. Theoretically, the change in curvature increases with reduction in the value of (*EI*), and therefore, the amount of damage can be obtained from the magnitude of change in curvature.

A simply supported beam having a uniform rectangular cross-section, can be simulated as a discrete model. The dimensions of the reinforced concrete beam used in this study are shown in Figure 1. The discretised model of the beam consisted of 55 equal length elements. Theoretically, three degrees of freedom i.e. translations in the *X* and *Y*-axis and rotation about the *Z*-axis, can be investigated. However, only the translational DOF in the *Y*-axis will be considered in this study. This was done so as to focus only on the pure bending modes of the beam under modal testing and neglecting the lateral bending and torsional modes.

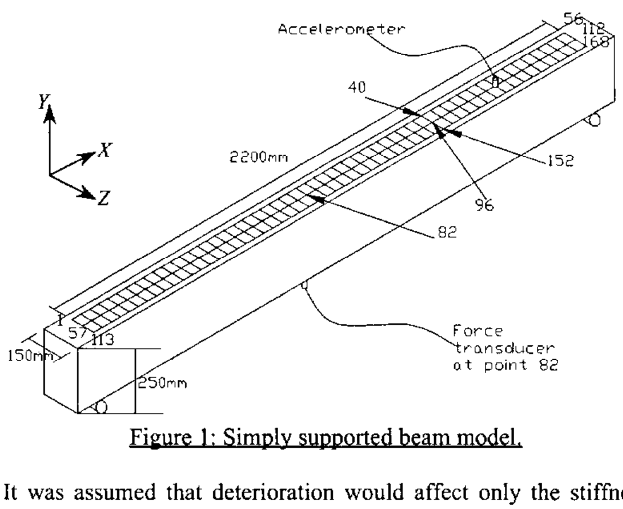


Figure 1: Simply supported beam model.

It was assumed that deterioration would affect only the stiffness matrix in this eigenvalue problem. Since modal analysis will return satisfactory information about the displacement mode shape of the translational DOF in the *Y*-axis, i.e. $[y_j]$, algorithms can be applied to investigate the stiffness changes occurring during the deterioration process in the beam. The discretised displacement mode shape is shown in Figure 2.

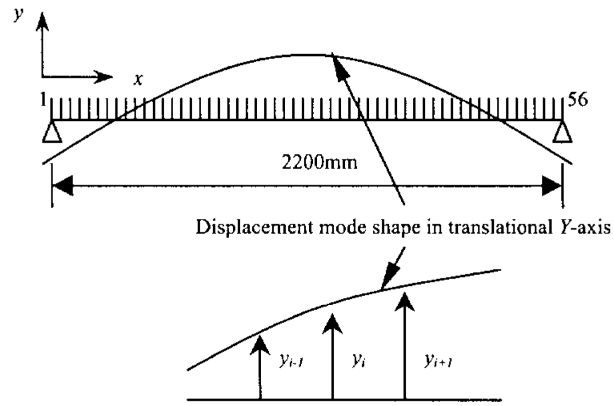


Figure 2: Displacement mode shape for discretised beam model

In vibration analysis, for a one-dimensional structure, the displacement mode shape is a function of the translational coordinates of the beam and is given as follows,

$$y = f(x)$$

In order to investigate the changes in displacement mode shape, the fundamental Newtonian differential was utilised. Since a discretised model of the beam was used in this investigation, the first differential can be approximated as follows,

$$y' = f'(x) = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$

This is known as the slope of the displacement mode shape.

However, in the discretisation process, the distance between two elements, h , was constant and fixed at 40mm in this study, thus

$$y' = \frac{y_{i+1} - y_i}{h} \qquad (2)$$

To further the mathematical derivation, the curvature of the displacement mode shape can be approximated as,

$$y'' = f''(x) = [\frac{(y_{i+1} - y_i)}{h} - \frac{(y_i - y_{i-1})}{h}]/h$$

$$y'' = \frac{(y_{i+1} - 2y_i + y_{i-1})}{h^2} \qquad (3)$$

To simplify the formulation, the distance between the two elements h, is taken as unity and this applies since the only variable for the discretised model, as shown in Figure 2, is displacement in the *Y*-axis. As such, curvature of the displacement mode shape, also refrred to as the Laplacian Operator (*LO*) is given as,

$$(LO)_i = y_{i+1} - 2y_i + y_{i-1} \qquad (4)$$

From previous research work, the *LO* was successfully applied to numerical data obtained from FE modelling and experimental data on structural component comprising of a homogenous material. However, when applied to non-homogenous and composite material such as reinforced concrete element, the operator was too sensitive and in certain cases did not yield meaningful results. Hence, a simplified Laplacian Operator (*SLO*) is proposed, which is the first difference of the discrete function of the displacement mode shape as given in equation (2), and rewritten as

$$(SLO)_i = y_{i+1} - y_i \qquad (5)$$

3. EXPERIMENTAL PROGRAM

A reinforced concrete beam with the dimensions shown in Figure 1 was cast. The beam was simply supported on steel rollers placed on concrete blocks. The top surface of the beam was marked with 168 grid points along three lines across the span of the beam which served as the measurement points for the modal test. The transfer functions were acquired through a signal analyser Initially, the transfer function spectrum within a 5kHz frequency span was obtained in order to locate roughly the resonant frequency peaks of all the flexural modes within the band. Subsequently, zooming within a 100Hz span of the resonant frequency peak of a particular mode was carried out. The measurements were made using a block size of 400 lines thus giving a resolution of 0.25Hz per spectral line. By using modal analysis software, the curve fitting process was performed on the transfer function spectrums obtained to extract the natural frequencies, damping ratios and mode shapes. A total of ten normal bending modes (NBM) were acquired in this manner. Throughout the modal testing process, the excitation point was permanently fixed at point no. 82. The accelerometer was moved from point to point until all the measurement locations were covered.

Prior to the application of load to induce cracking in the beam, a set of modal data was obtained. This served as the datum for comparative purposes in this study. Then, a 20mm deep saw-cut was introduced across the beam at soffit points no. 40, 96 and 152 as shown in Figure 1. This was to create a point of initiation of a single crack when load was applied. The beam was then subjected to a concentrated line load at a position coincident with the saw-cut until the required severity of cracking was achieved.

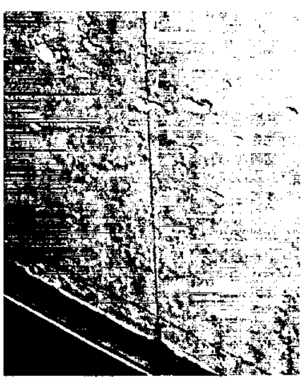


Figure 3: Saw-cut at the soffit of the beam.

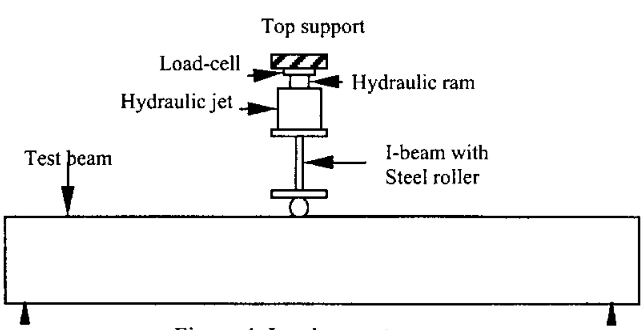
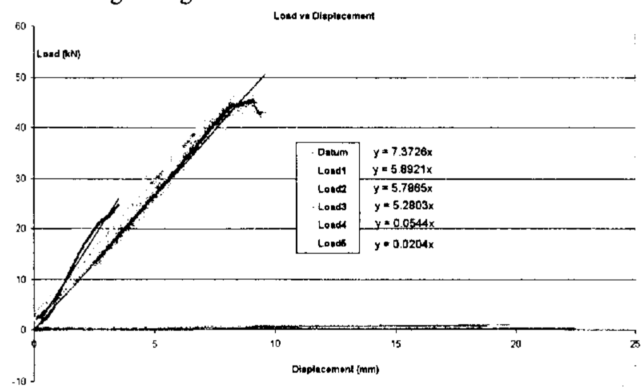


Figure 4: Load test set-up.

The load and mid-span displacement were measured using a load cell and two transducers respectively, and the data recorded in real-time using a digital data acquisition system. The load test set-up is shown in Figure 4. Crack-gauges were fixed across the crack region to monitor the crack width. The load on the beam was applied until the required crack width was obtained. The beam was then unloaded and modal test conducted to obtain the modal parameters for the cracked beam. This was repeated for subsequent loading stages until the beam failed.

4. RESULTS AND DISCUSSION

The load against displacement during the load test is plotted in Figure 5 and the slope provides a measure of the flexural stiffness. It shows that the stiffness of the beam decreases as the intensity of the cracking damage increases.



Stage	*Gradient*	*Relative Stiffness*
Datum	7.3726	1.000
Load1	5.8921	0.799
Load2	5.7865	0.785
Load3	5.2803	0.716
Load4	0.0544	0.007
Load5	0.0204	0.003

Figure 5: Relative stiffness for each stage of loading.

Results of the measured crack widths are given in Figure 6. The first crack was visible at the first loading stage. This was followed by a load intensity to induce a crack width just slightly more than the serviceability requirement of BS8110, that is at the second loading stage. Increasing the load further gave crack widths which exceeded the serviceability requirements, and in practice would imply a level of severity requiring rehabilitation or loading restrictions. Finally at a crack width of about 1mm the load carrying capacity was diminished.

Stage	*Gauge 1*	*Gauge 2*	*Average Crack*
Datum	0.000	0.000	0.000
Load1	0.068	0.079	0.073
Load2	0.425	0.341	0.383
Load3	0.734	0.665	0.700
Load4	1.101	1.050	1.076
Load5	3.574	2.695	3.135

Figure 6: Unrecoverable crack width for each stage of loading.

The natural frequency of a structure has a direct relationship with the structural stiffness. For an undamped vibrating system with mass m, the relationship can be written in the following form:

$$\omega_n \propto \sqrt{\frac{EI}{m}} \qquad (6)$$

However, the frequencies acquired from modal testing are the global natural frequencies, and serve as a good indicator for detecting the occurrence of damage, without providing information on the location of the deteriorated part in the beam. The modal parameters obtained from modal testing, namely natural frequencies, damping ratio and percentage damping are summarised in Figure 7.

Mode	Datum f_r	Datum ζ	Datum %	Load1 Stage f_r	Load1 Stage ζ	Load1 Stage %	Load2 Stage f_r	Load2 Stage ζ	Load2 Stage %
NBM1	75.08	2.53	3.37	78.09	2.34	3.00	75.89	2.04	2.69
NBM2	499.83	1.83	0.37	498.25	4.42	0.89	469.28	6.44	1.37
NBM3	856.93	3.60	0.42	841.73	3.92	0.47	786.03	4.64	0.59
NBM4	1316.90	5.05	0.38	1302.77	5.15	0.40	1243.29	29.20	2.35
NBM5	1797.48	6.21	0.35	1774.03	2.68	0.15	1692.92	8.96	0.53
NBM6	2310.23	1.17	0.05	2268.91	8.58	0.38	2135.24	9.54	0.45
NBM7	2821.89	6.54	0.23	2786.55	13.12	0.47	2597.51	3.85	0.15
NBM8	3361.69	11.43	0.34	3298.65	12.54	0.38	3145.35	16.50	0.52
NBM9	3893.46	14.47	0.37	3833.97	17.58	0.46	3543.67	16.48	0.47
NBM10	4411.49	15.70	0.36	4353.10	15.73	0.36	3960.19	15.99	0.40

Mode	Load3 Stage f_r	Load3 Stage ζ	Load3 Stage %	Load4 Stage f_r	Load4 Stage ζ	Load4 Stage %	Load5 Stage f_r	Load5 Stage ζ	Load5 Stage %
NBM1	74.59	1.79	2.40	72.64	1.44	1.98	69.60	1.22	1.75
NBM2	466.16	10.73	2.30	454.98	8.80	1.93	455.83	15.61	3.42
NBM3	774.14	5.40	0.70	749.94	4.81	0.64	650.81	5.99	0.92
NBM4	1195.84	29.24	2.45	1095.13	12.08	1.10	1018.91	7.38	0.72
NBM5	1666.79	10.18	0.61	1622.33	7.51	0.46	N/A	N/A	N/A
NBM6	2060.65	11.01	0.53	1957.57	13.11	0.67	1719.73	8.97	0.52
NBM7	2478.33	6.46	0.26	2382.27	8.76	0.37	2047.65	10.96	0.54
NBM8	3082.52	15.02	0.49	2944.28	12.69	0.43	2899.13	16.96	0.59
NBM9	3329.54	15.54	0.47	3179.79	10.92	0.34	3031.37	19.42	0.64
NBM10	3798.25	18.24	0.48	3328.90	12.41	0.37	3207.59	16.03	0.50

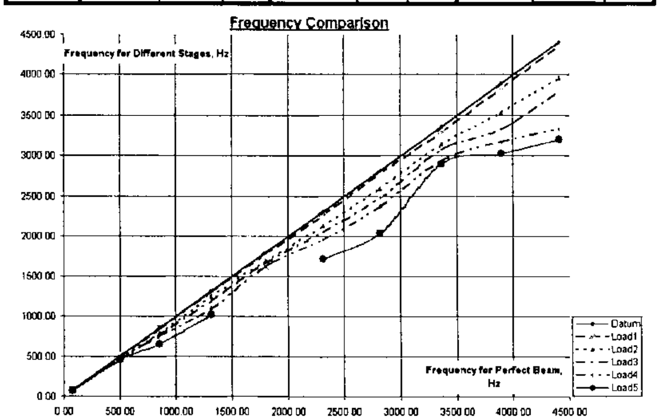


Figure 7: Comparison of natural frequencies for loading stages.

The natural frequencies of the beam suffered a reduction when cracking was induced. It was also apparent that the higher the degree of damage, the higher the drop in natural frequency. It should be noted that the natural frequency of mode NBM5 of Load5 stage could not be obtained since the node point of the mode coincided with the excitation point of the vibrating source. Nevertheless, the results acquired from the tests is consistent with the relationship given in equation (6). Hence, a drop in natural frequency when compared to datum readings obtained prior to induced damage, is an indicator of the occurrence of damage in the beam. It can also be observed that there is a deviation from a linear relationship after the second loading stage especially for the higher modes. The deviation is more eminent as the crack width increases. Damping results do not exhibit any consistent trend and for the purpose of this study is not suitable as a damage indicator.

Laplacian Operator/Simplified Laplaçian Operator (LO/SLO)

Mode shape data as damage location indicator has caught much attention lately. Modal parameters extracted can be categorised into two groups, namely local response parameter such as mode shape data and global response parameters such as natural frequency and damping. Normally, global response parameters techniques require previous known data set such as from a perfect beam or FE simulation data sets for comparison purposes. By using local response parameter, i.e. mode shape in this investigation, no prior knowledge of the undamaged structure or FE simulation data sets are required. It operates solely on the mode shape from the damaged structure, and therefore it is very practical when applied to actual structures.

Figure 8 shows the displacement mode shape acquired using the transfer function method (TFM) as carried out in the experimental program. The displacement mode shape for normal bending mode 1 (NBM1) is shown by averaging all the data acquired from the 168 measurement points for the various loading stages. It can be seen that the curvature increases with the severity of the crack.

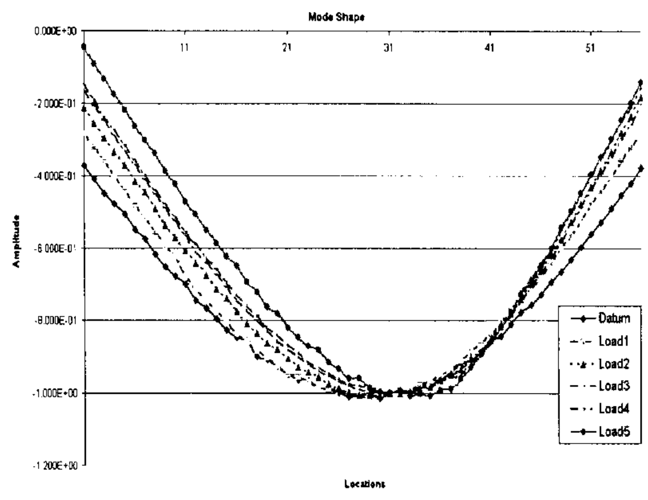


Figure 8: Displacement mode shape for NBM1.

The use of the Laplacian operators on mode shape data to identify the damaged location in the beam are illustrated in Figure 9 to 18. Generally the plots of the LO values reveal that they are irregular with no definite pattern. This is in contrast to the results obtained

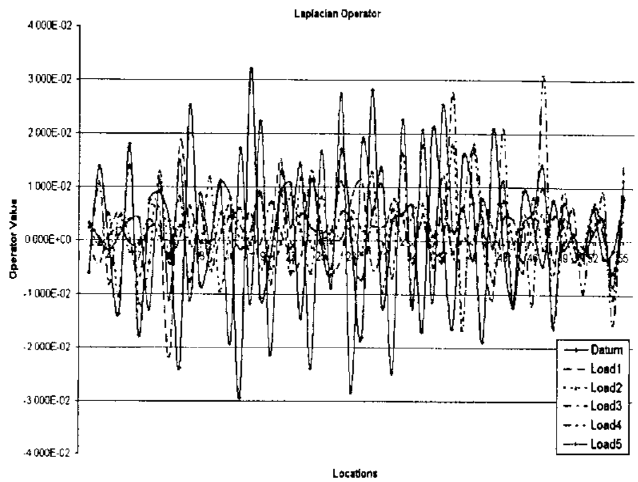


Figure 9: *LO* for NBM1.

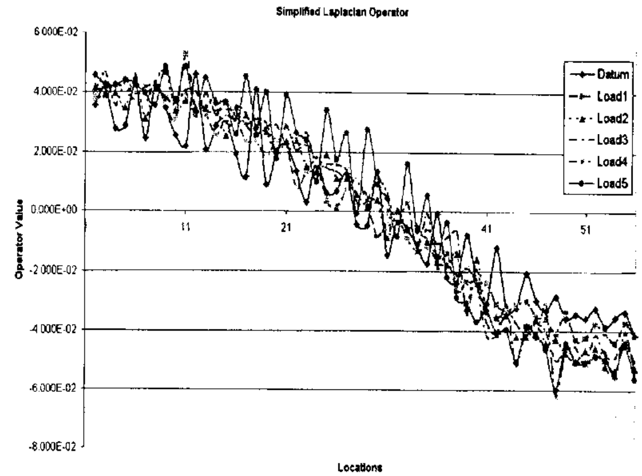


Figure 10: *SLO* for NBM1.

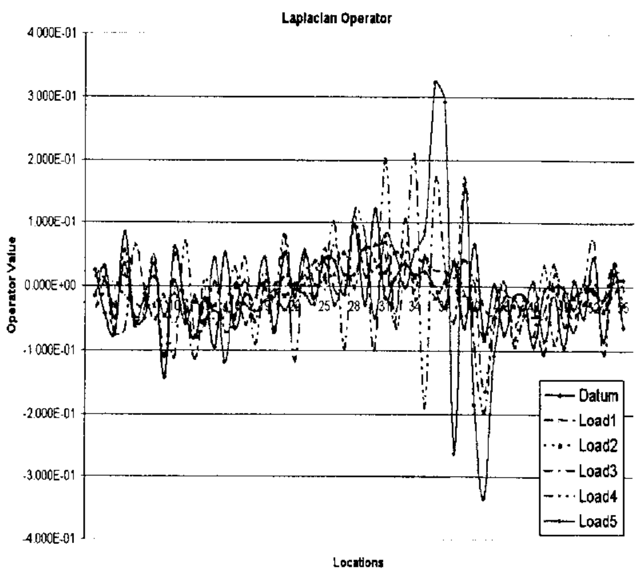


Figure 11: *LO* for NBM3.

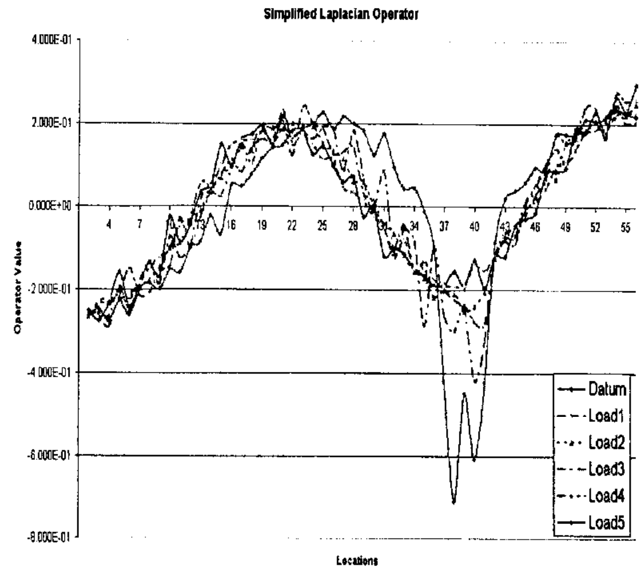


Figure 12: *SLO* for NBM3.

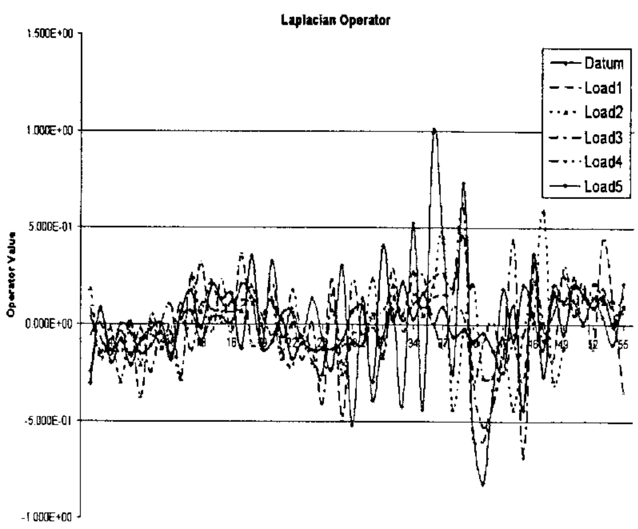


Figure 13: *LO* for NBM6.

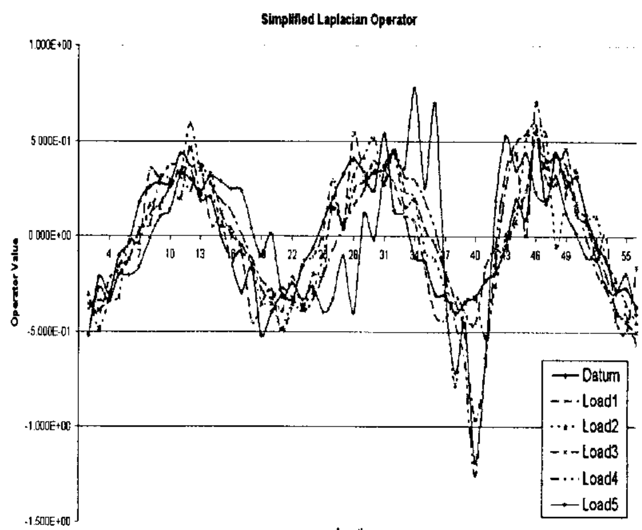


Figure 14: *SLO* for NBM6.

by *C. P. Ratcliffe* [1] whereby the plots were smooth and a noticeable anomaly would only occur at the damage location. The reasons for this discrepancy is probably due to reinforced concrete being a non-homogenous material, the influence of the support condition and the effect of shear deformation in the beam. For lower modes such as NBM1, both the operators provide no information about the damage location in the beam as shown in Figure 9 and 10. Due to the low complexity of the displacement mode shape, the length of the damaged region is relatively short in comparison to the distance between node points and would not influence the mode shape significantly. Moreover, the existence of anomalies contributed by material non-homogeneity, support condition and shear deformation would be too dominant. Therefore, it is anticipated that the Laplacian operators for the lower modes is incapable of detecting the local change in structural stiffness.

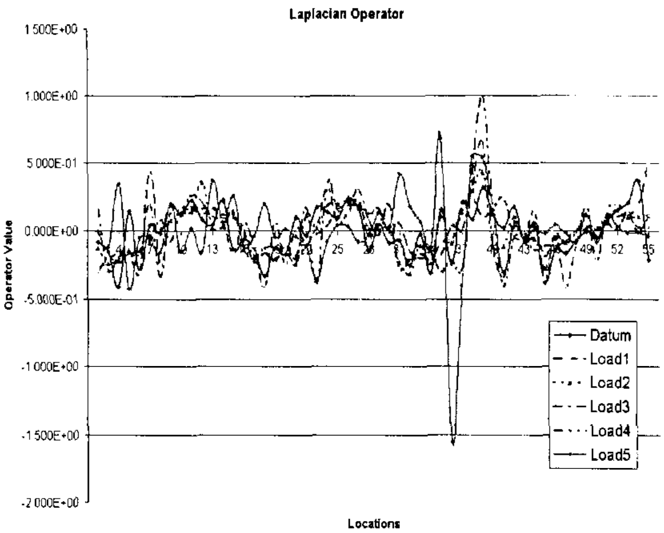


Figure 15: *LO* for NBM8

However, as the mode increases, such as mode NBM3 and NBM6, the mode shape becomes more complex comprising of more node points along the span. Consequently, the aspect ratio between the length of the damage zone and the distance between node points would be higher. Hence, the local change in structural stiffness occurring within the damage zone would more significantly affect the mode shape. As a result, the values of both the Laplacian operators would register higher anomalies with respect to change in structural stiffness occurring within or close to the damage zone as illustrated in Figures 11 to 14. It is apparent that the *LO* is too sensitive when applied to the data set for mode NBM3 as in Figure 13. However, *SLO* detect the damaged location in the beam accurately that is at point 40 as in Figure 12. Nevertheless, the damaged location can be predicted precisely by both the operators when the damage is severe as in the case of the final loading stage.

For the higher modes such as NBM6, the location of the defect is evident as shown in the values of the operators in Figure 13 and 14. Again, the anomalies as exhibited by the *LO* is more prominent which tend to overshadow the results at the damage zone. In contrast, the use of the *SLO* clearly indicate the damage zone and the localised change in structural stiffness can be detected as early as loading stage 3. This shows that a higher frequency mode is a better indicator to detect the deterioration process at an earlier stage. Lower frequency modes such as NBM1 and NBM2 are suitable as damage indicators only if the level of severity is

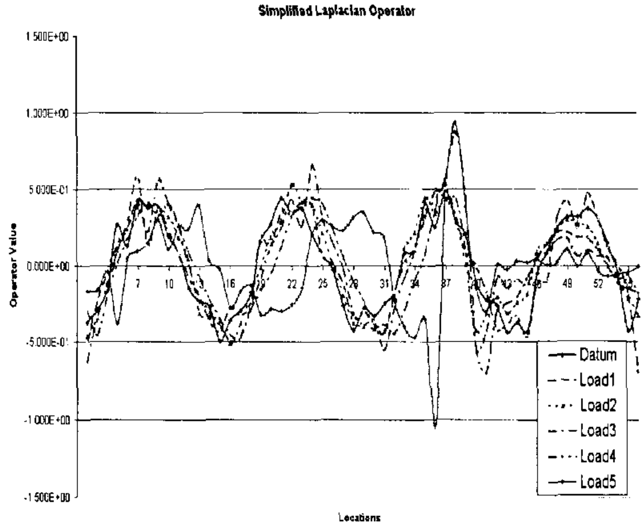


Figure 16: *SLO* for NBM8

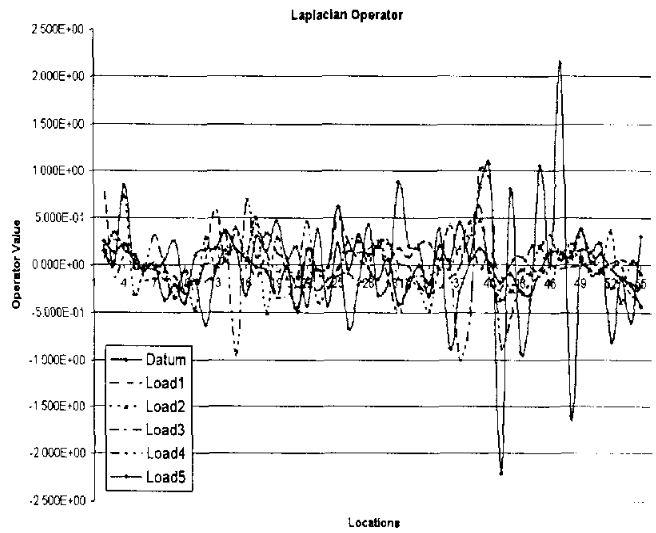


Figure 17: *LO* for NBM10

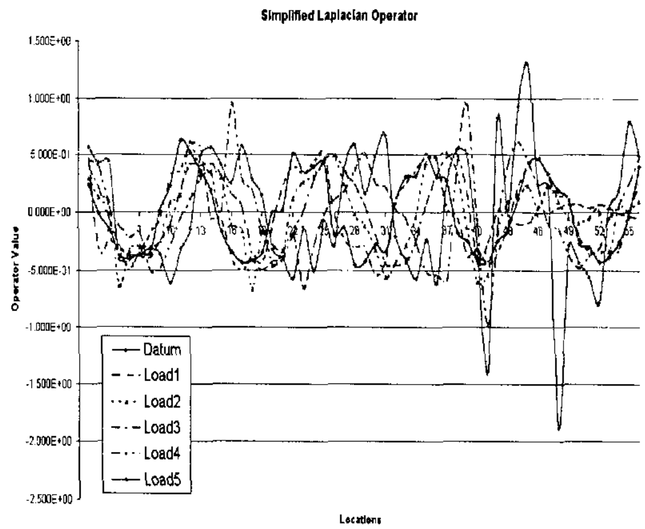


Figure 18: *SLO* for NBM10

high. This trend becomes obvious for NBM8 and NBM10 as shown in Figure 15 to 18. Furthermore after the fourth loading stage the occurrence of another crack at point 37 in addition to the main crack at point 40, introduced another anomaly in the plot. This was also sensed by the *SLO* for NBM3, NBM6 and NMB8 as shown in Figure 12, 14 and 16 respectively.

5. CONCLUSIONS AND RECOMMENDATION

From this study, it is shown that the use of modal parameters namely natural frequencies and mode shapes for crack detection in reinforced concrete structural elements is viable and practical. It provides an alternative technique to conventional methods for structural assessment purposes and the approach is also more integrated. The shifts in natural frequencies would indicate the presence of the defect and also correlate with the residual strength since the relationship of the frequency shift is linear with changes in the stiffness.

Curvature of the displacement mode shapes obtained by using Laplacian operators can be used successfully to detect the crack location in the structure. However, in this study it is shown that the use of the simplified Laplacian operator is more appropriate for the modal data obtained from the reinforced concrete beam. From the results of this study, it was also apparent that the use of higher frequency modes could sense and identify the crack location for less severe cases. Lower frequency modes can be used when the severity of cracking is high.

The Laplacian operators operate solely on the data set acquired from the damaged structure. No comparative data set is required for purpose of determining the damage location. More studies and observations are required to obtain damage indices to indicate the level of severity of the deterioration in the structure.

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REFERENCES

[1] **Ratcliffe C. P.**, *Damage Detection using a Modified Laplacian Operator on Mode Shape Data*, Journal of Sound and Vibration, 204 (3), pg. 505-517, January 1997.

[2] **Chondros T. G.** and **Dimarogonas A. D.**, *Identification of Crack in Welded Joints of Complex Structure*, Journal of Sound and Vibration, 69, pg. 531-538, 1980.

[3] **Masoud S., Jarrad M. A.** and **Al-Maamory M.**, *Effect of Crack Depth on the Natural Frequency of a Prestressed Fixed-fixed Beam*, Journal of Sound and Vibration, 214(2), pg. 201-212, 1998.

[4] **Ismail F., Ibrahim A.** and **Martin H. R.**, *Identification of Fatigue Cracks from Vibration Testing*, Journal of Sound and Vibration, 138, pg. 305-317, October 1989.

[5] **Ostachowicz W. M.** and **Krawczuk M.**, *Analysis of the Effect of Crack on the Natural Frequencies of a Cantilever Beam*, Journal of Sound and Vibration, 150(2), pg. 191-201, October 1990.

[6] **Pandey A. K., Biswas M.** and **Samman M. M.**, *Damage Detection from Changes in Curvature Mode Shape*, Journal of Sound and Vibration, 145(2), pg. 321-332, May 1990.