Sensitivity of Natural Frequencies to Composite Effects in Reinforced Concrete Elements

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During the last two decades, investigations on the dynamic properties of structural elements have been the subject of numerous research works. The primary reason for this is the awareness and interest in using dynamic testing techniques for the purposes of health monitoring and damage detection for engineering structures. The dynamic properties of any structural element are governed by the relationship of the material properties and the boundary conditions. For steel, the dynamic properties relate to steel element properties, which are assumed to be the same under different load conditions, and its boundary conditions. For concrete elements, such as plain concrete, the dynamic properties are related to the behavior of the concrete element, which will have different behaviors under different load conditions, and its boundary conditions. Reinforced concrete (RC) elements have a composite effect due to the presence of different materials to form the RC elements. Therefore, in order to simplify the modeling of the mechanical behavior ofRCelements, the boundary conditions are assumed to be the same under different conditions for the purpose of this present study

The stiffness of RC elements under different loading conditions is a function of steel behavior, concrete behavior, and the interaction between them. Although many studies have been done in the field on the mechanical behavior of RC elements, research work in this area is ongoing due to the complexity arising from the composite nature of the materials [1]. Thus, an investigation on the relationship between the dynamic and static properties of RC elements should take into consideration the behavior of each material under different conditions, the interaction between steel bar and concrete, and its influence on the overall element stiffness. The relationship between the dynamic and static properties, i.e., the natural frequency and the stiffness of the structural elements, is expressed in the equation for transverse free vibration of a simply supported Bernoulli-Euler beam given by:

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where f is the natural frequency, n is mode number, m is mass per unit length, and L is span length. Rewriting Equation 1, and representing the flexural rigidity, EI by the symbol K, the expression below is obtained:

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implying that a change in flexural rigidity causes a change in natural frequency.

For elements comprising either steel or plain concrete only, K is a function of their respective material properties, where the boundary conditions effect is ignored. For a composite element, such as RC, the stiffness, K, is a function of both the individual material properties as well as the interaction between the materials. Thus, for RC, the following equation applies:

$$K_{RC} = K_{concrete} + K_{steel} + K_{bond}.$$

Concrete stiffness is dependent on its behavior under different loading conditions. For RC beams, the concrete stiffness is represented by its behavior in compression aswell as in tension as given below:

$$K_{concrete} = K_{tension} + K_{compression}.$$

The steel stiffness for an RC beam (i.e., K_{steel}) is the stiffness of steel under tension loading conditions only since the steel is normally positioned in the tension zone.

When load is applied, the concrete stiffness in both tension and compression will change according to the loading level and its behavior under compression or tension loading action. Cementitious materials are characterized by a softening response,which can vary depending on its strength in compression and tension. Experimental results show that these materials exhibit brittle behavior in tension and inelastic deformation accompanied by damage effects in compression [1]. Steel stiffness will be governed by the stress-strain relationship obtained fromtensile tests. The interacting force in the interface element between the steel and concrete elements has zero value when no load is applied, but increases correspondingly when load is gradually applied to resist the slipping of the steel bar.

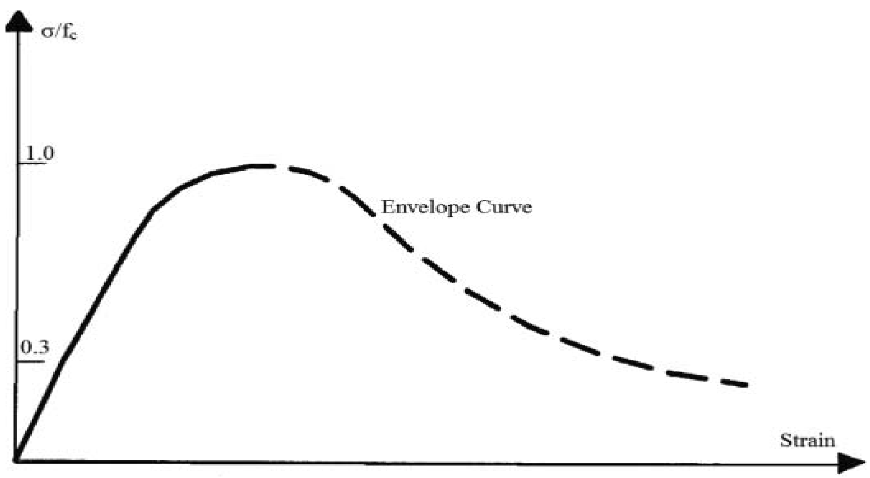

Fig. 1. Response of concrete to uniaxial compression.

Concrete Behavior

Concrete is a material with a grossly heterogeneous internal structure. It consists of inter-aggregate particles embedded within a binding pastemade of cement andwater. The presence of micro-cracks in the transition zone between the cement paste and the aggregate prior to any load application can be viewed as a source of weakness in the structure of the concrete [2]. Many of these micro-cracks are caused by segregation, shrinkage, and thermalmovements in the mortar. Some microcracks may develop during loading because of the difference in stiffness between the aggregate and the mortar. The gradual growth of these micro-cracks with further loading contributes to the nonlinear behavior of concrete [3].

Concrete can behave as either a linear or a nonlinear material, depending on the nature and the level of the induced stresses. Many experimental studies on the behavior of concrete under uniaxial and multiaxial loading have been performed.

The stress-strain relationship for concrete subjected to uniaxial compression is nearly linear elastic up to about 30% of its maximum compressive strength (f_{c}), as shown in Figure 1. For stresses beyond this point, there is a gradual increase in curvature up to about 0.75 f_{c} to 0.9 f_{c}, whereupon it bends more sharply and approaches the peak point at f_{c}. Beyond this peak, the stress-strain relationship has a descending trend until crushing failure occurs at some ultimate strain, $\sum_u$ [4]. The stress level of about 30% of f_{c} has been termed the onset of localized cracking and has been proposed as a limit of elasticity [5].

For concrete under uniaxial tensile stress, the stress-strain relationship hasmany similarities to that of uniaxial compression. Generally, at a stress level less than 60% of the tensile strength, the creation of new micro-cracks is negligible. So, this stress level will correspond to a limit in elasticity. Beyond this level of stress, the growth of micro-cracks begins. The direction of crack propagation for uniaxial tension is transverse to the stress direction. The growth of every new crack will reduce the available load-carrying area, and this reduction causes an increase in the stresses at critical crack tips. The failure in tension is caused by a few bridging cracks rather than by a number of cracks, as in the case for compressive states of stress [6].

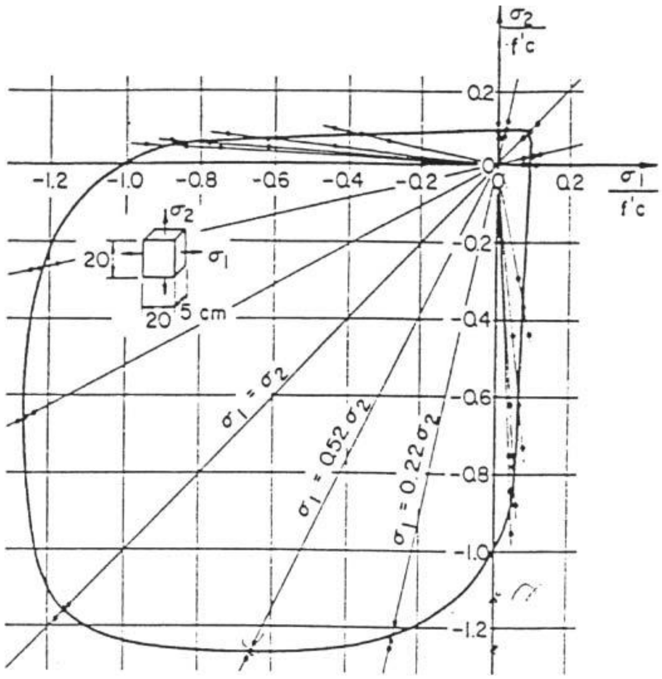

Fig. 2. Failure envelope of concrete in biaxial stress space

Under different combinations of biaxial loading, concrete exhibits strength and stress-strain behavior somewhat differently from that under uniaxial conditions. For biaxial compression states, the maximum strength increases approximately 25% at a stress ratio of 0.5 and 16% at a stress ratio of 1.0, as shown in Figure 2 [7]. Under biaxial tension, concrete exhibits a constant strength [7], or a slight increased in tensile strength compared to values obtained under uniaxial loading [8]. Under biaxial compression–tension, the compressive strength decreases almost linearly as the applied tensile stress is increased.

In plain and reinforced concrete structures, cracking is not a perfectly brittle phenomenon and experimental evidence shows that the tensile stresses normal to a cracking plane are gradually released as the crack width increases. For RC structures, where the behavior is characterized by the formation of many closely spaced cracks, the nature of the stress release is further complicated by the restraining effect of the reinforcing steel. After cracking, the concrete stresses drop to zero and the steel carries the full load. The concrete between cracks, however, still carries some tensile stresses. This ability of concrete to share the tensile load with the reinforcement is termed the tension stiffening phenomenon [3].

From the above-mentioned behavior of concrete, it can be summarized that concrete in compression behaves linearly up to 30% of its compressive strength while maintaining constant stiffness. Beyond this point, it exhibits nonlinear behavior and experiences a decrease in stiffness. The decrease in stiffness is rapid until the peak point at $f_{\_}$ $_c$. Beyond this peak, the stressstrain curve has a descending part until crushing failure occurs at a point of ultimate strain, $\sum_u$. Correspondingly, concrete in tension behaves linearly up to 60% of its maximum tensile strength. Beyond this point, micro-cracks will grow and join up together in the form of tension cracks, and the behavior of concrete becomes nonlinear. This will lead to a decrease in the stiffness of concrete in tension during the unloading stage. Stresses will be transferred from the steel bar to the concrete between cracks, and this will cause an increase in the element stiffness. This ability of concrete to share the tensile load with the reinforcement is termed the tension stiffening phenomenon.

Steel Behavior

Compared to concrete, steel is a much simpler material to represent. Its stress-strain behavior can be assumed to be identical in tension and compression. In RC members, reinforcing bars are normally long and relatively slender and therefore they can be assumed to be capable of transmitting axial forces only. The uniaxial stress-strain behavior of reinforcement is represented by an elastic-linear work hardening model. Steel will have linear behavior until yielding. Before the yielding of steel, there is no change in steel stiffness during the unloading stage. Beyond the yield point, however, steel will exhibit nonlinear behavior resulting in a decrease in steel stiffness at the unloading stage.

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