DETECTION OF DEFECTS IN REINFORCED CONCRETE BEAMS USING MODAL DATA

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ABSTRACT

Analyses were done on a control reinforced concrete beam and four reinforced concrete beams with various extent of defects in the form of voids and load-induced cracking. The voids were artificially created by incorporating known percentage volumes of polystyrene balls in the concrete mix at predetermined locations along the beams. Flexural cracks were induced at two different locations and at predetermined levels of loading to give different severity.

Modal analysis was used to produce eigenvalues and eigenvectors so that natural frequencies and mode shapes can be generated for all the beams. The mode shape equation for each beam was obtained by using a curve fitting technique for the generalized solution of the form,

$$Y(x) = C_1 \sinh \lambda x + C_2 \cosh \lambda x + C_3 \sin \lambda x + C_4 \cos \lambda x$$

Local flexural stiffness at each coordinate point was derived by substituting the regressed data at that point and using the centered-finite-divided-difference formula. The results of the first derivative of the local flexural stiffness for the reinforced concrete beams with voids were compared with those obtained from a similar beam with cracks in the middle. The results were also compared with a similar beam with cracks on the right side of the beam.

NOMENCLATURE

EI	flexural stiffness
ω	natural frequency
ρ	density of concrete
A	cross-sectional area
L	length of the beam
W	width of the beam
H	height of the beam
Y	displacement in y-direction
x	coordinate location
$f^{(iv)}$	fourth derivative

1. INTRODUCTION

The periodic structural condition monitoring of reinforced concrete structures is necessary to ensure that they provide a continued and safe service condition. Current assessment procedures usually rely on visual inspection and location-dependent methods. This study proposes the application of modal analysis to evaluate the stiffness of reinforced concrete beams with defects. The presence of defect causes localised changes to flexural stiffness and thus by observing and investigating the distribution obtained, the location and severity can be established.

Most dynamic tests for the observation of structures apply the first natural frequency as the most convenient parameter [1]. However, changes in stiffness can be due to increase in the modulus of elasticity E with time because of concrete hardening [2]. Casas proposed a more reliable method of surveillance of concrete structures through the characteristics of the natural frequencies and mode shapes [3].

There have been significant efforts to detect the location of defects using one or more of the modal properties [4-14]. The most easily observable change is the reduction in natural frequencies, and most investigators use this feature in one way or another [15-18]. Varying success is reported using the change in modal damping, and some work has been reported on the use of change of mode shape to detect the location of damage [19-21]. Locating the damage is not a simple matter. Shahrivar and Bouwkamp presented the finite element and experimental data for a scale model of an offshore platform and concluded that the fundamental mode shape was more sensitive to damage than the fundamental vibration frequency [22]. Ratcliffe presented a technique for damage detection by applying a Laplacian operator to discretely measured mode shape obtained from the damage beam [20]. Maeck et al used a technique to predict the damage location and intensity directly from measured modal displacement derivatives [23].

2. BACKGROUND

2.1 Transverse Vibration of a Prismatic Beam

The Equation for free transverse vibration of a uniform beam, in one dimension is

$$\frac{d^4 Y}{dx^4} - \lambda^4 Y = 0 \qquad (1)$$

where $\lambda^4 = \frac{(\rho A \omega^2)}{EI}$. (2)

The general solution[24,25] given in Equation. (1) may be written as

$$Y(x) = C_1 \sinh \lambda x + C_2 \cosh \lambda x + C_3 \sin \lambda x + C_4 \cos \lambda x \quad (3)$$

2.2 Curve fitting

The curve fitting technique used in this paper is nonlinear regression, namely the Marquardt-Levenberg algorithm. It is a weighted average of Newton's method and the Steepest Descent method for nonlinear systems of equations. The weight is biased toward the Steepest Descent method until convergence is detected, at which time the weight is shifted toward the more rapidly convergent Newton's method [26,27].

2.3 Numerical Differentiation

The centered finite-divided-difference formula

$$f^{(iv)} = \frac{f(x_{i+2}) - 4f(x_{i+1}) + 6f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{h^4} \quad (4)$$

was utilized for estimating the fourth derivatives in this study.

3. EXPERIMENTAL WORK

The transfer function technique was adopted for modal testing on one control, two test beams with voids and a further two test beams with load induced cracking. The dimensions of the reinforced concrete beams were 150 mm wide and 250 mm deep. The beams were simply supported across an effective span of 2200mm on concrete blocks. Modal analysis was used to produce eigenvalues and eigenvectors so as to determine the natural frequencies and mode shapes of the beams.

The mode shape data obtained were curve-fitted into the generalized solution of mode shape equations given by Equation 3, by using non-linear regression. For the beams with defects it was approximated that the generalized solution still applies. The λ^4 is derived by applying Equation 1 and the centered-finite-divided-difference formula on the regressed data. The next step was to derive the local flexural stiffness at each coordinate point. This was done by rearranging Equation 2. Here again, for the beams with defects, it was approximated that the generalized equation applies.

3.1 Beams with Voids

The changes in the first derivative of the local flexural stiffness were evaluated from modal data obtained from the control beam without any defect and two test beams with voids. These voids were formed by incorporating precast mortar blocks with 40% by volume of polystyrene balls in the mix. The blocks were located at mid-span of the beams. Details of the dimensions and location of the blocks are given in Table 1.

	Dimensions of Mortar Blocks ($LxWxH$ mm)
L10	220 x 90 x 90
L10x2	220 x 90 x180

Table 1. Void Dimensions

3.2 Beams with Cracks

Similarly, the changes in first derivative of the local flexural stiffness of the two beams with cracks were derived using modal data. On one of the beams, a crack was introduced at mid-span of the beam (x = 1080 mm) while on the other, a crack was introduced on one side of the beam (x = 1560 mm). Prior to application of load to induce cracking in the beams, a set of modal data was obtained and act as datum. The beams were then subjected to a concentrated line load at the corresponding positions where the crack is to be introduced. The magnitude of the load applied depends on the level of severity required based on the crack widths. Crack widths were measured at the soffit level of the beam using crack gauges. Details of the crack widths and location of the cracks are given in Table 2. Upon reaching the level of severity, the beam was unloaded and modal test was performed to generate a set of modal data for that particular loading stage. This was repeated for subsequent loading stages until the beams failed.

	Beam 1 (crack at x =1080mm)	Beam 2 (crack at x =1560mm)
Load 1	0.080	0.038
Load 2	1.310	0.383
Load 3	-	0.700
Load 4	-	1.076

Table 2. Crack width after each loading in mm

4. RESULTS AND DISCUSSION

4.1 Beams with Voids

The mean EI values for the beams with voids are lower than for the control beam. The more severe the voids the lower the values. The standard deviation increases with increasing extent of the voids. The summary of the results are given in Table 3.

The normalised EI for the control beam and beams with voids are given in Figure 1. It is seen that the more severe the defect, the higher the normalised values.

Figure 2 shows the graphs of the first derivatives of local flexural stiffness values for Mode 1 of the control beam and the test beams with voids. The local flexural stiffness values were obtained by applying Equation(1) and subsequently Equation(2) to the regressed data at each coordinate point. Generally the derivatives of local flexural stiffness for the beams with voids demonstrate higher rates of change compared with values for the control beams.

4.2 Beams with Cracks

The mean and standard deviation of EI values for the datum readings of both beams are quite similar. For the beam with a crack in the middle, it is seen that the mean EI drops with increasing crack severity as evident in Table 4. A similar trend was observed for the beam with a crack on one side as indicated in Table 5. Judging from the values of relative stiffness, it is apparent that the crack in the middle causes a greater loss in stiffness as compared to the case for which the crack is on one side, for approximately the same level of damage severity induced. This is to be expected since the results were compiled for the first mode where the maximum response amplitude occurred at the crack location and would return a greater change in local flexural stiffness.

The normalised EI for the datum and the various loading stages for the beam with cracks in the middle and on one side are given in Figure 3 and 5, respectively. Both exhibit identical trends with higher normalised values with increasing severity. However, the pattern of the plots of the values for the beam with a crack on one side deviated from that obtained for the datum. This is apparent after loading stage 1 where the crack widths exceeded the limit of serviceability. It is also evident from the plots that the point where the curve starts to deviate is almost coincident with the location of the induced crack.

The values of the first derivatives of local flexural stiffness for both the beams are shown in Figure 4 and 6. Generally, it can be observed that the rate of change increases with increase in crack widths. The plots resemble horizontal S-shape curves with the exception of the results obtained after the first loading stage for the beam with the crack on one side (x = 1560mm). Again, the deviation from the shape obtained for datum was evident and the curve is flatter due to the fact that the crack location shifts away from the middle.

	Mean of EI	Std Dev. of EI	Rel. Stiffness
Control	1.077×10^7	3.980×10^2	1.0
L10	7.198×10^6	5.691×10^2	0.67
L10x2	5.248×10^6	6.781×10^2	0.49

Table 3. Mean, Standard Deviation of EI and Relative Stiffness values for Control and Beams with Voids

	Mean of EI	Std Dev. of EI	Rel. Stiffness
Datum	8.364×10^6	6.319×10^2	1.0
Load 1	4.455×10^6	8.922×10^2	0.53
Load 2	2.665×10^6	9.820×10^2	0.32

Table 4. Mean, Standard Deviation of EI and Relative Stiffness values for Beam with Crack at x = 1080mm

	Mean of EI	Std Dev. of EI	Rel. Stiffness
Datum	7.888×10^6	6.590×10^2	1.0
Load 1	7.592×10^6	5.703×10^2	0.96
Load 2	6.323×10^6	13.07×10^2	0.80
Load 3	5.599×10^6	13.59×10^2	0.71
Load 4	4.856×10^6	15.20×10^2	0.62

Table 5. Mean, Standard Deviation of EI and Relative Stiffness values for Beam with Crack at x = 1560mm

It is perceived that the severity of the damage, may it be due to the voids or crack, bears a relationship with the rate of change of the first derivatives of EI for the beams. In order to quantify the rate of change of the first derivative, the curves were fitted with a third order polynomial function. This was applied to the results for both beams with the exception of the curves for the second loading stage and beyond for the beam with the crack on one side. The results for the coefficients of x^3 together with their degree of fit (r^2) are shown in Table 5. It shows that the absolute values for all beams increase with increasing severity of the defects.

A damage index based on the relative stiffness of the defect beams as given in Table 3 to 5 is proposed. From the plot shown in Figure 7, it is evident that the severity can be established based on the changes in the local flexural stiffness. However, the sensitivity is dependant on the type of defect present, as evident by the higher values of the fitted coefficient obtained for the cracked beams as opposed to the beams with voids.

	Coefficient of x^3	r^2
Beams with Voids		
Control	-3152.694	0.99758
L10	-4639.778	0.99652
L10x2	-5140.022	0.99875
Beam with Crack at x = 1080mm		
Datum	-4132.824	0.99899
Load 1	-9954.033	0.99307
Load 2	-13677.783	0.98651
Beam with Crack at x = 1560mm		
Datum	-6456.661	0.99449
Load 1	-7504.004	0.98513

Table 5. Coefficient of x^3 and Degree of Fit for all Beams

5. CONCLUSIONS

The procedure of applying curve fitting using the generalized solution model and fourth order centered finite-divided-difference to the modal data yields the local flexural stiffness for the control and defect reinforced concrete beams. The curves of normalised EI for these beams are almost flat indicating constant values of EI along the span of the beam irrespective of the presence of damage.

However the changes in the first derivative of the local EI is more appropriate as an indicator of damage in the beams. The shifts are dependent on the level of severity and the trend obtained for the two types of defect considered in this study is consistent. Furthermore, the shape is a characteristic of the location of the defect zone having an S-shape for defect lying in the mid-span of the beam. Divergence from the S-shape is indicative of the defect lying in a region away from the mid-span and located at the point where the deviation becomes conspicuous. In order to quantify the degree of damage, a relationship based on the fitted coefficient of the first derivative of the local EI obtained from modal data is proposed.

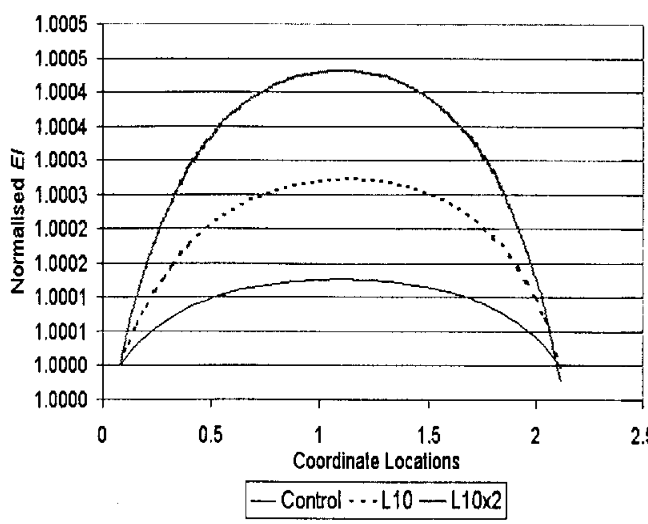

Figure 1. Normalised EI for Control and Beams with Voids

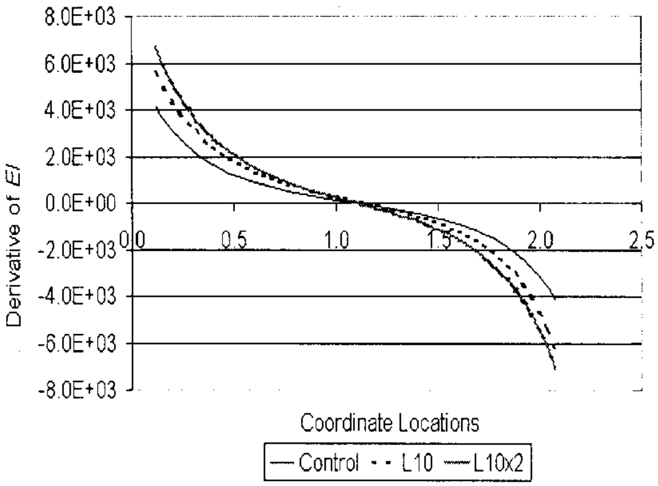

Figure 2. First Derivatives of Local EI for Beams with Voids

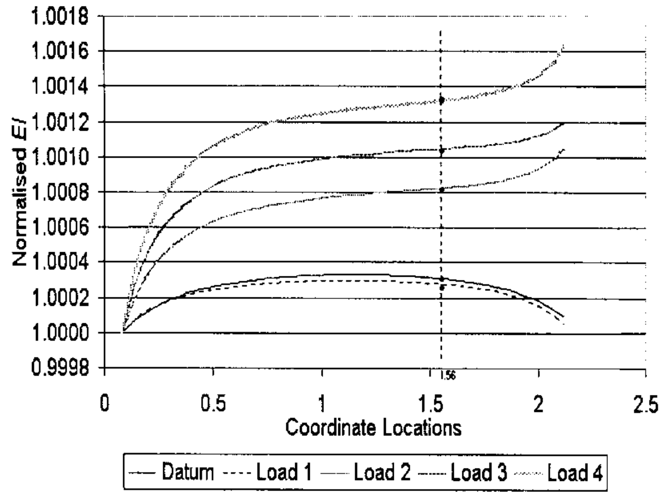

Figure 5. Normalised EI for Beam with Cracks at x = 1560mm

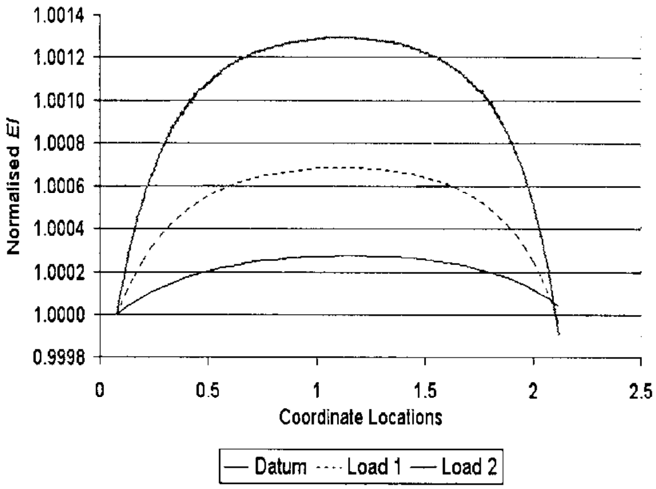

Figure 3. Normalised EI for Beam with Crack at x = 1080mm

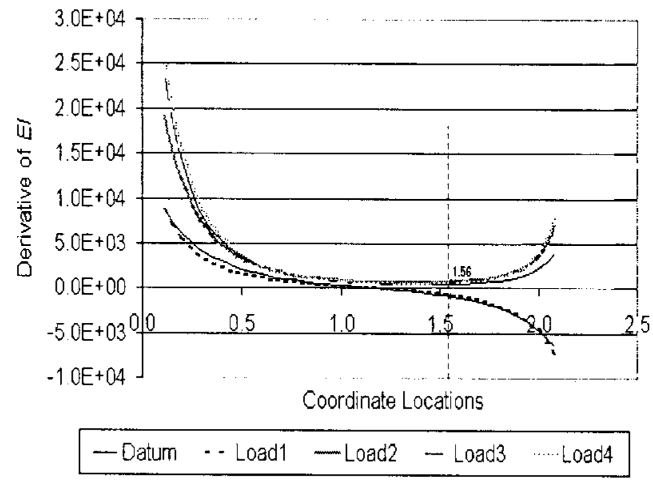

Figure 6. First Derivatives of Local EI for Beam with Crack at Location x = 1560mm

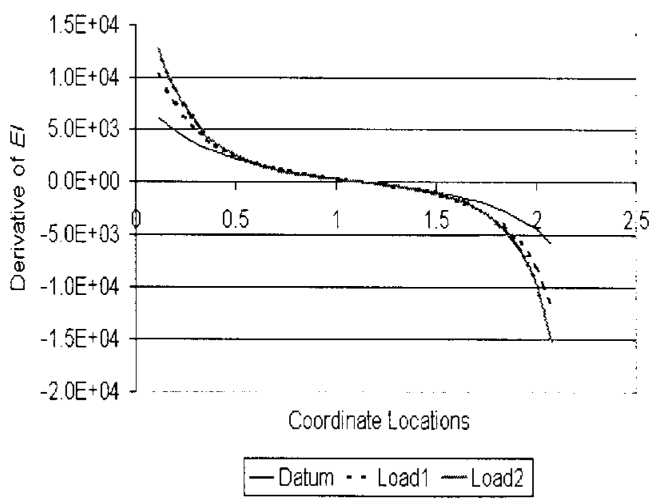

Figure 4. First Derivatives of Local EI for Beam with Crack at Location x = 1080mm

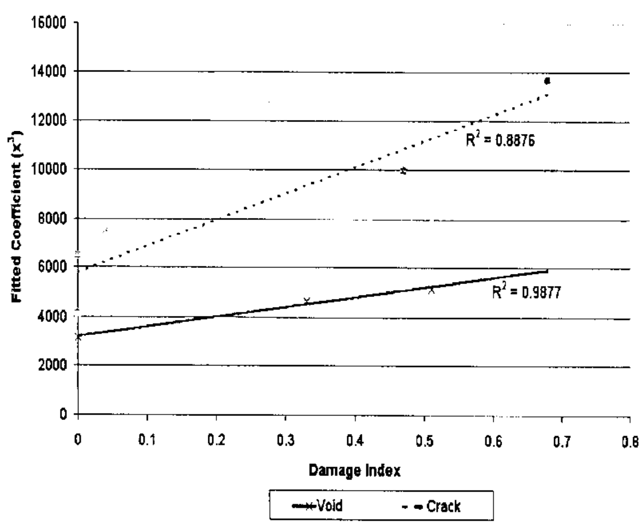

Figure 7: Effect of damage on fitted coefficient of derivative

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