



## The Effect of Support Condition on Dynamic Parameters

M. M. Fayyadh and H. Abdul Razak

*Dept. of Civil Engineering, Faculty of Engineering, University of Malaya, 50603, KL, Malaysia*

Bridge girders are normally supported on rubber bearings which are subjected to deterioration during the life span of the bridge. Early deterioration of the bearings results in decrease of its stiffness while advanced deterioration can cause total damage of the bearings, whereby the bridge girders will be directly supported on the structural element below it. This paper reports an investigation on the effect of support condition on the dynamic parameters of a simply supported beam. Experimental modal analysis was conducted on the beam under different support stiffness with new and old rubber bearings and without rubber bearings. The natural frequencies and mode shapes were acquired from the test and compared using various algorithms as indicators for the state of the supports. From the results, the flexural natural frequencies showed good sensitivity to the support conditions which makes it a feasible tool for health monitoring purposes. This is applicable to all the bending modes with the exception of mode 3 which showed an opposite trend. The lower modes had higher sensitivity to the differences in support condition and it can be concluded that mode 1 was more reliable in assessing the boundary conditions.

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### 1. Introduction

Many engineering structures when exposed to various external loads such as earthquakes, traffic, explosion and vibration during their lifetime suffer damage and deterioration. This seriously affects their performance and may even lead to catastrophic structural failures. Health monitoring of engineering structures using dynamic testing has gained a lot of interest over the last few years. The basic idea behind this approach is that modal parameters namely natural frequency, mode shape, modal damping, are functions of physical properties of structures such as mass, damping and stiffness as well as boundary conditions. Therefore, any change in the physical properties or boundary conditions will in effect cause detectable changes in the modal parameters. There are many previous studies related to the use of natural frequency or mode shape for damage identification. [1-6]

One of the major concerns in the long term performance of bridges is the deterioration of boundary conditions as a result of stiffness change in bridge bearing pads over time. As such, the monitoring of the boundary conditions is very important in ensuring timely maintenance before any serious damage occurs to the structure. There have been some studies done on the effect of boundary conditions on the dynamic properties of the structure. W. Dai *et. al.* [7] investigated the effect of stiffness of the rubber pads on dynamic characteristics of base isolated bridge. The results show direct relationship between rubber stiffness and frequencies. It shows whenever the rubber stiffness increases, frequencies increase as well. Thomas *et. al.* [8] investigated the effect of support stiffness and damping on measured modal frequencies and damping ratios. The effect of support system on both modal frequencies and modal damping were illustrated on two different types of structures. The effect of support condition on measurement of modal parameters was investigated by Wolf [9] and Carne [10]. They found that there is direct relationship between the support stiffness and the measured modal parameters.

The main objective of this study is to investigate the effect of support stiffness on the dynamic properties of a structure. Correspondingly the sensitivity of each fundamental bending frequency to changes and different support conditions will help to identify and establish the appropriate damage indicator. This is achieved by obtaining the reduction in natural frequency and by comparing mode shapes using the Modal Assurance Criteria (MAC).

## 2. Damage Indicators

Modal parameters are used as damage indicators or dynamic properties monitoring tool. The change in natural frequency and mode shape are used as indicators to compare the results for different support conditions.

The first indicator is the Frequency Reduction Index FRI due to reduction in Natural Frequency. Defining the natural frequency reduction factor  $R$  as

$$R = \left(1 - \frac{f_{i,d}}{f_{i,c}}\right) \cdot 100\% \quad (1)$$

where  $f_{i,c}$  and  $f_{i,d}$  are the natural frequency at  $i$ th mode for control and damaged beam, respectively; and utilizing Eq. (1) will lead to the Frequency Reduction Index (**FRI**) which is an indicator that utilizes only the natural frequency:

$$FRI = 2R \quad (2)$$

The second indicator is the Mode Shape Reduction Factor MSRI, by means of Modal Assurance Criteria MAC. This method is used to ascertain the configuration errors between the experimental mode shapes and the eigenvectors predicted from the finite element model called the modal assurance criterion (MAC) Ewins [11]. It is also a correlation between any two sets of mode shape. The correlation for the  $i$ th element is given by the following formula

$$MAC^* = \frac{\left|\sum_{i=1}^n \varphi_{i,c} \cdot \varphi_{i,d}\right|^2}{\left(\sum_{i=1}^n \varphi_{i,c} \cdot \varphi_{i,c}\right) \left(\sum_{i=1}^n \varphi_{i,d} \cdot \varphi_{i,d}\right)} \quad (3)$$

and  $\varphi_{i,c}$  and  $\varphi_{i,d}$  are the mode shapes at  $i$ th mode for control and damaged beam respectively.

Utilizing the concept in the previous paragraph, the Mode Shape Reduction Index (**MSRI**), an indicator that utilizes only mode shapes, is defined by the following formula

$$\text{MSRI} = (1 - \text{MAC}^*) \cdot 100\% \quad (4)$$

### 3. Experimental Work

The experimental work was conducted on a simply supported concrete beam reinforced with minimum requirement in accordance to ACI-318-08 [12]. The span length of each beam was 2200 mm with cross sectional area of 150 mm by 250 mm and reinforced with 2 Nos. 12 mm high-yield steel bars as longitudinal reinforcement and 8 mm mild steel bars as shear reinforcement with spacing of 100 mm. Figure 1 shows the beam's reinforcement and dimensions details. Table 1 shows the cases of the support conditions used in this study. Figure 2 shows different support conditions of the beam. In order to compare the stiffness of new rubber and old rubber, compressive strength tests were carried out on different samples of the rubbers. Figure 3 shows compressive load against deflection curves. Specimen 1 is the new rubber, while Specimen 2 is the old rubber. The test showed that the old rubber have lost a portion of its stiffness.

### 4. Results and Discussion

After the test beam was cast, experimental modal analysis was conducted on the beam. In order to simulate the effect of different support stiffness on the modal parameters, different rubber bearings were utilized. The support condition adopted in this study are namely 'weak' stiffness where the beam was supported by old rubber (OR-OR) with stiffness of 3 MN/m, 'control' stiffness supported by new rubbers (NR-NR) with stiffness of 10MN/m, and 'high' stiffness supported directly on steel supports(SS-SS). The first seven bending modes from force vibration testing using transfer function were acquired.

#### 4.1 Effect on Frequencies

The effects of the support stiffness on the natural frequencies of the first six bending modes are shown in Figure 4. The results showed increase in natural frequencies with the increase in support stiffness, with the exception of mode 3 where the trend was opposite. Thus mode 3 is a good indicator to verify the effect of boundary conditions rather than the effect of structural element stiffness on the dynamic parameters. In order to study the sensitivity of each mode to the change in support stiffness, the Frequency Reduction Index FRI index was applied on the results. A comparison was conducted based on the control support stiffness (NR-NR). The FRI results are illustrated in Table 2. The results show that mode 1 is the most sensitive to change in support stiffness followed by mode 2. All the other five modes showed the same sensitivity to the change in support stiffness.

#### 4.2 Effect on Mode Shapes

The effect of different support stiffness on the mode shape is illustrated by utilizing the first seven bending mode shapes. In order to study the effect, comparisons with the control support stiffness was carried out using the Mode Shape Reduction Index MSRI. Figure 5 shows the MSRI values for different modes under different support stiffness. The results show that mode 1 is the most sensitive to the change in support stiffness for both cases. For the case of high support stiffness (SS-SS),

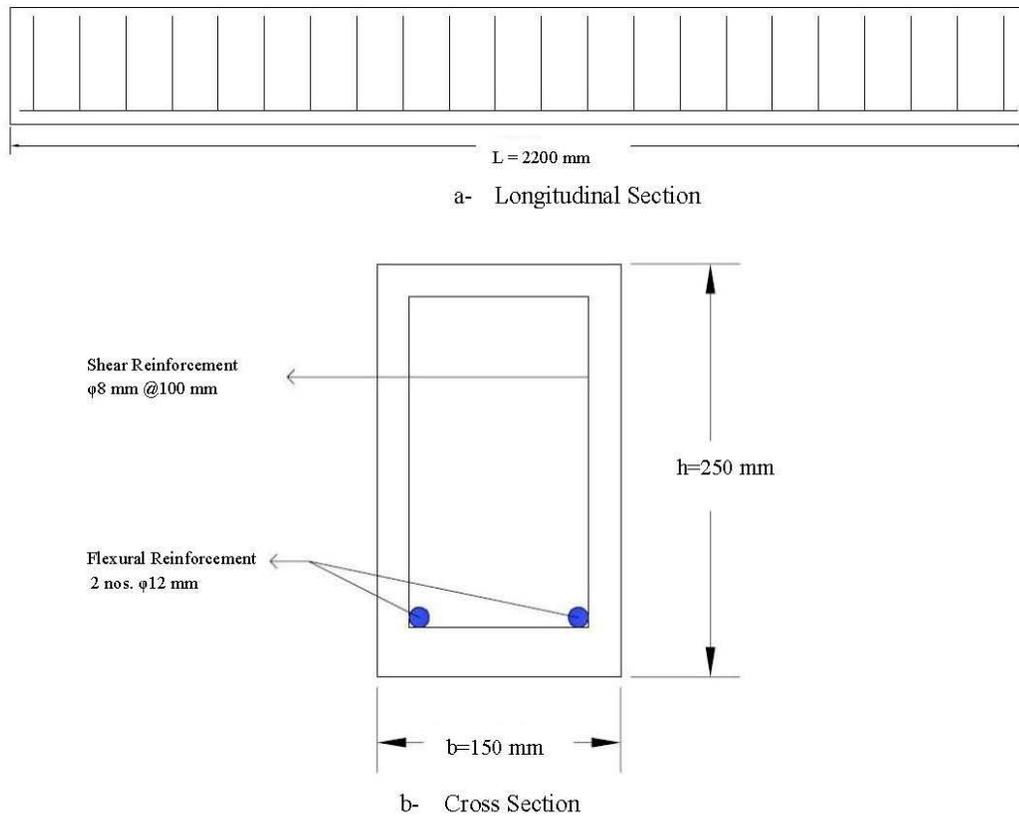
modes 2 and 3 also exhibited significant sensitivity to the change in support stiffness, while all the remaining other modes show very low and similar sensitivity. The weak stiffness support case shows that modes 2, 3 and 4 are not sensitive to the change in support stiffness, while modes 5, 6 and 7 have low and similar sensitivity to the change in support stiffness.

## Conclusion

The bending natural frequencies showed good sensitivity to the changes in support conditions, thus making them feasible indicators for monitoring the health status of the supports. This is applicable for all the bending natural frequencies except for mode 3 which showed an anomaly. Bending mode shapes 1 and 2 showed high sensitivity to the difference in the support conditions. Mode 1 gave the highest sensitivity to any change in support condition, which makes it more reliable as an indicator compared to the others. Although, mode 3 showed a trend opposite to the other modes, it can be used as a good indicator when both effects of boundary condition and structural stiffness on the dynamic properties are present. This study enhanced the understanding and supports previous work on the use of dynamic parameters for monitoring purposes.

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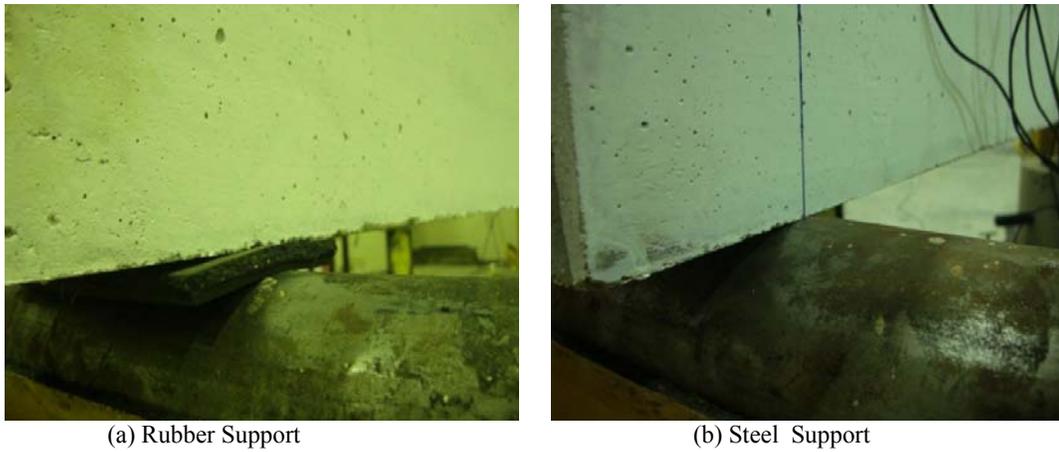
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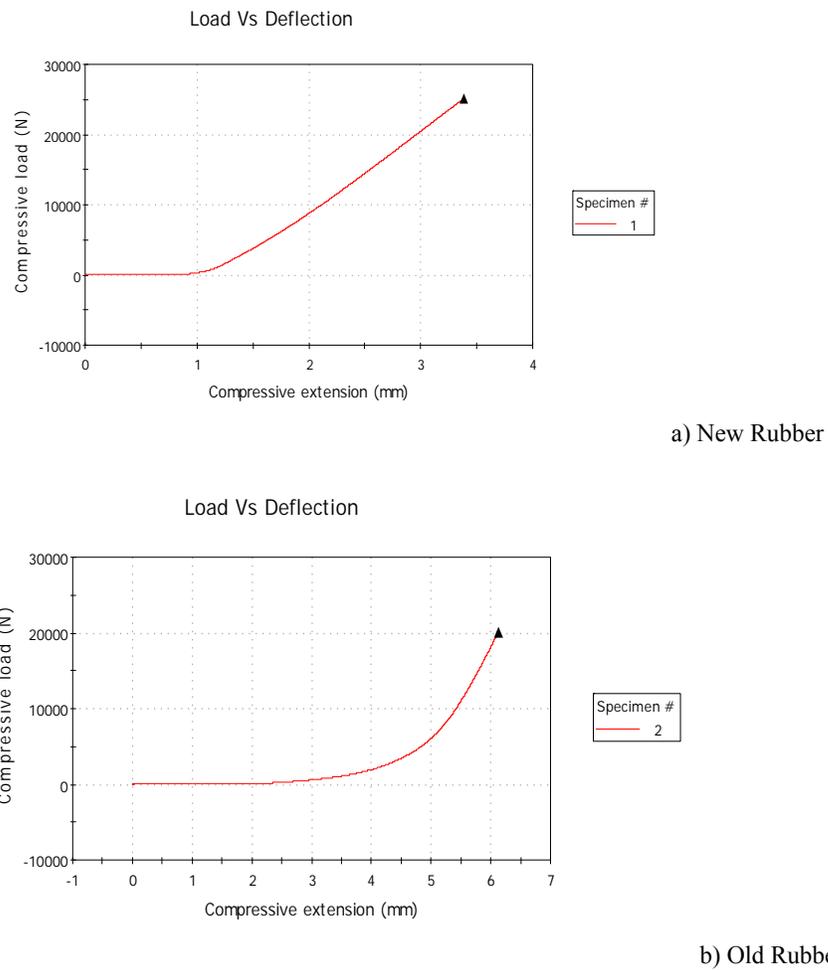
**Figure 1.** Details and dimensions of a Reinforced concrete beam.

**Table 1.** Support condition cases used in this present study.

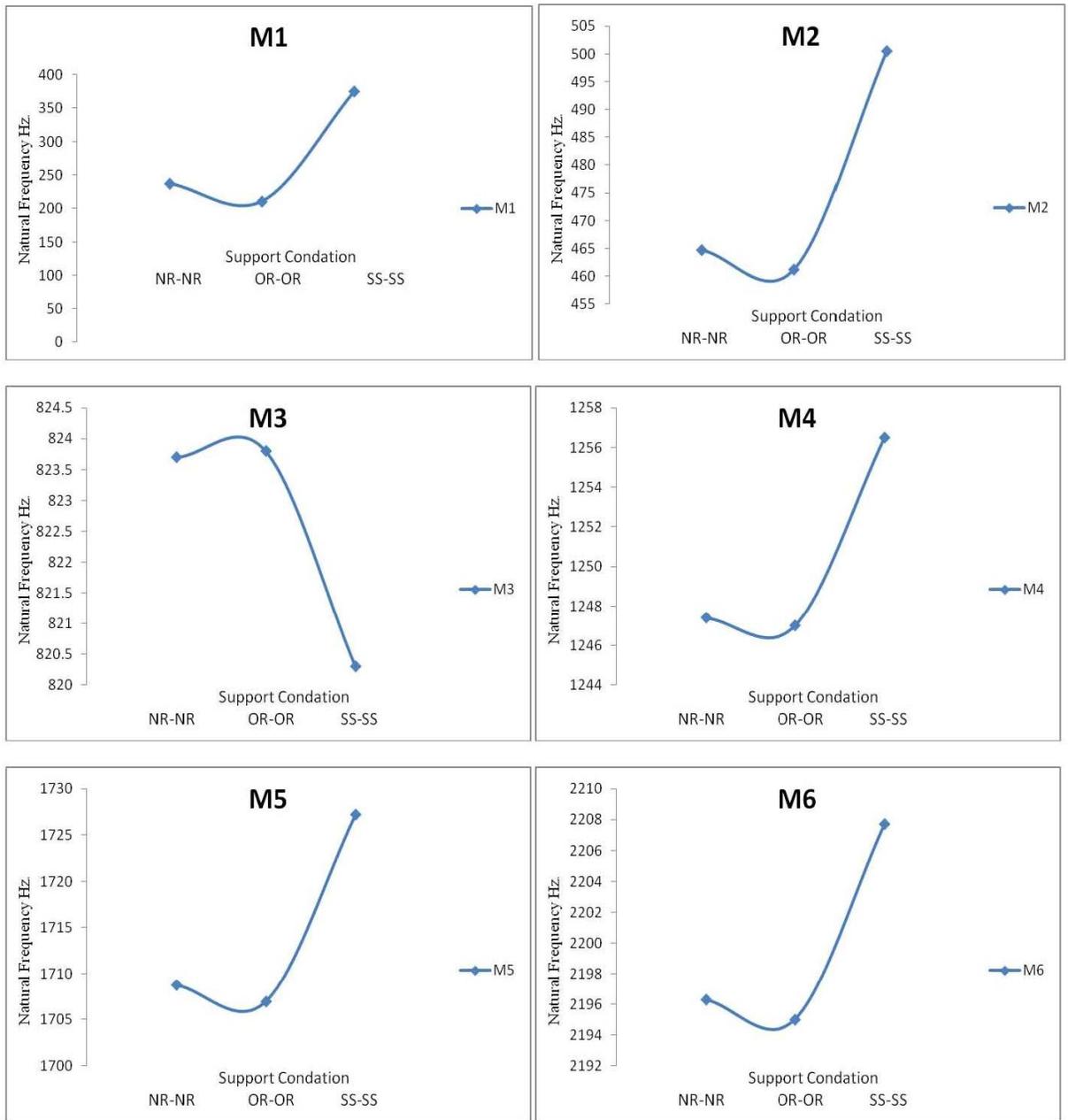
| Support Conditions | Left support condition | Right support condition | Left support stiffness | Right support stiffness |
|--------------------|------------------------|-------------------------|------------------------|-------------------------|
| NR / NR            | New rubber             | New rubber              | 10 MN/m                | 10 MN /m                |
| OR / OR            | Old rubber             | Old rubber              | 3 MN/m                 | 3 MN/m                  |
| SS / SS            | Steel Support          | Steel Support           | $\infty$               | $\infty$                |



**Figure 2.** Different support conditions



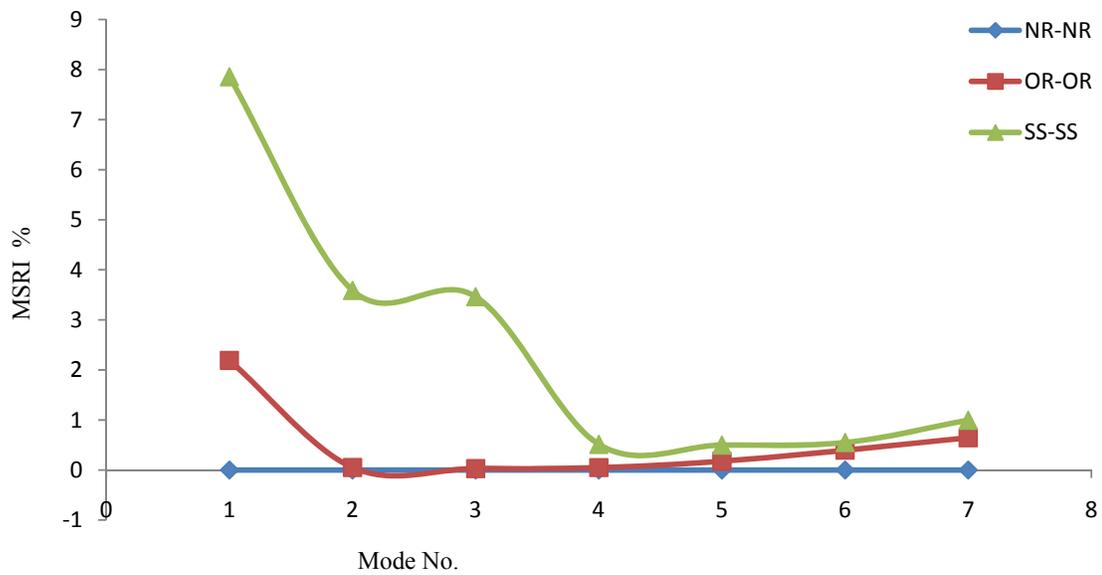
**Figure 3.** Load vs compressive extension curves for both rubber types.



**Figure 4.** First six natural frequencies with different support conditions

**Table 2. FRI values for different support stiffness compare to the control support stiffness**

| Support Condaton | FRI Index % |        |        |        |        |        |        |
|------------------|-------------|--------|--------|--------|--------|--------|--------|
|                  | Mode 1      | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 |
| OR / OR          | -11.17      | -0.75  | 0.01   | 0.02   | 0.07   | 0.09   | -0.04  |
| SS / SS          | 57.82       | 7.70   | -0.41  | 0.73   | 1.08   | 0.52   | 0.38   |



**Figure 5. MSRI values for different support stiffens based on control supt support stiffness**