Associated computational plasticity schemes for nonassociated frictional materials

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The stress–strain behavior of geomaterials such as clay, sands, rock, and concrete can often, at least to a first approximation, be accounted for in terms of simple linear elastic/perfectly plastic models. Indeed, although a very large number of more complex models have been developed over the years, the simple linear elastic/perfectly plastic models remain widely used in engineering practice. Such models comprise three key ingredients: an elastic law, a yield criterion, and a flow rule. These components are all relatively straightforward to either measure or estimate. Regarding the flow rule, one often assumes a flow potential of the same functional form as the yield function, with a dilation coefficient replacing the friction coefficient of the yield function. The Mohr–Coulomb and Drucker–Prager models are often used as a basis for this approach. In this way, the excessive dilation predicted by the flow rule associated with the yield function may be adjusted to a more realistic magnitude.

Although the specification of an arbitrary flow rule in principle is straightforward, the deviation from associativity has a number of far reaching consequences. From a mathematical point of view, the introduction of a nonassociated flow rule usually leads to a situation where the governing equations, at some characteristic stress state, go from being elliptic to being hyperbolic. Physically, this loss of ellipticity indicates an instability where a homogeneous mode of deformation gives way to a localized deformation pattern defined by one or more shear bands [1–6]. Such localized modes of deformation give rise to a number of complications related to mesh dependence, internal length scales, and so on. Secondly, and more seriously, it has frequently been reported that numerical solutions to boundary value problems involving nonassociated constitutive models are much more difficult to obtain than in the case where the flow rule is associated [7–10]. These complications have a tendency to be more pronounced for high (but realistic) values of the friction angle and the degree of nonassociativity. Similarly, for fixed material parameters, one usually observes a degradation of the performance as the number of finite elements in the model is increased.

These facts motivate a closer look at the physical origins of nonassociated flow rules and the numerical methods used to solve problems of frictional plasticity. In the following, inspired by the micromechanical origins of friction and its modeling in terms of plasticity theory, a new approach to computational plasticity for frictional (and generally nonassociated) materials is presented. The resulting scheme essentially approximates the original nonassociated problem as one of associated plasticity. Consequently, all the well-established numerical procedures for standard associated plasticity are applicable with little modification.

The paper is organized as follows. The governing equations are briefly summarized in Section 2 before the new approach of approximating general nonassociated plasticity models in terms of equivalent associated ones is presented in Section 3. In Section 4, two different solution algorithms are presented. The first one is a slight modification of the common fully implicit scheme by Simo and his coworkers [11, 12]. Secondly, following recent work of the authors [13–16], we formulate the governing equations in terms of a mathematical program. For certain yield criteria, notably Drucker–Prager, the resulting discrete programs may be solved very efficiently using a second-order cone programming solver, SONIC, recently developed by the authors. Next, in Section 5, the consequences of nonassociativity in terms of the ultimate load bearing capacity are discussed before the new numerical schemes are tested on some common boundary value problems in Section 6. These problems also highlight the consequences of nonassociated flow rules in terms of localization of deformations. Finally, conclusions are drawn in Section 7.

Matrix notation is used throughout with bold uppercase and lowercase letters representing matrices and vectors, respectively, and with T denoting the transpose.

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