The Non-divergence Newton-Based Load Flow Method in Ill Conditioned system

A.Shahriari, H. Mokhlis, A. H. A. Bakar, M. Karimi, M. M. Aman

Abstract— This paper implements the Optimal Multiplier Load Flow Method (OMLFM) in polar coordinate form to calculate low voltage solution and maximum loading point of system in ill conditioned system. OMLFM modifies the direction of state variables (buses voltage and phase) by using optimal multiplier in order to the convergence of load flow equations in ill conditioned system. The privilege of OMLFM emerges in keeping dimension of load flow jacobian matrix constant. While another method such as continuation and homotopy methods change the framework of jacobian matrix based on predictor and corrector steps in term of increasing load demand. Actually, the calculation process of SSSM is based on standard Newton Raphson load flow method. The validation of OMLFM for maximum loading point as ill-conditioned system is shown by testing IEEE 57 bus test system. Furthermore, the 13 bus radial transmission system is tested to with verify OMLFM for ill conditioned system includes high R/X ratio lines and the weak interconnection.

Index Terms— Optimal Multiplier Load Flow Method, low voltage solution, optimal multiplier, ill conditioned system
.

1 Introduction

Economic, Environmental, and technical problems such as the difficulty in construction of new transmission lines, new generation plants, and the raising load demand have let power system to operate near to its limit capacity[1, 2]. This is emerged in approaching to Maximum Loading Point (MLP), transmission lines with high R/X ratio and bus connections have a very high resistance and very low impedance that causing the power system becomes ill-conditioned system [3]. Therefore, chance of the voltage collapse phenomenon is increased due to power system operation in the instability zone [4]. Although, voltage collapse is a dynamic problem, it can be considered as a static issue if the power systems parameters change slowly [5]. The change in these parameters corresponds to a small load increase in power system. Hence, a set of static nonlinear algebraic equations of the power flow equations is considered for the voltage stability study [6]. In this context, a several approaches based on power flow equations have been introduced to determine load flow solutions in ill conditioned system. This case forces the power flow equations to have Low Voltage Solution (LVS) or solution Type-1 as unstable equilibrium operation point. Under this condition, the original power flow solutions as interesting power flow solution is called High Voltage Solution (HVS) [7].

In this regard, two main approaches are have been presented in order to find power system low voltage solutions, the Optimal Multiplier Load Flow Method (OMLFM) and path follow ing methods (PFM) [7, 8]. PFM utilizes trajectory of load demand enhancement in PV curves to detect MLP and beyond it [9]. This method is powerful and robust for detecting LVS [10]. However, PFM needs a large number of iteration and any prior information of the direction of load increasing to converge [7]. Also, PFM could not handle ill–conditioned and unsolvable condition in appropriate manner [11]. The PFM is classified as the Continuation methods (CM) and the Homotopy methods (HM) [9, 11, 12]. In contrary of CM, OMLFM does not need to several iterations and any previous information of direction of load demand increasing to calculate LVSs [13]. Because, OMLFM is based on Newton Raphson Load Flow Method (NRLFM) using optimal multiplier [13, 14]. The optimal multiplier as accelerator damper converges the load flow equations in ill conditioned systems [15].

This paper utilizes the OMLFM in polar coordinate to find LVS at Maximum loading point, high R/X ratio and the weak interconnection as ill-conditionding. The polar coordinate compares with rectangular coordinate is system is more appropriate for power flow solution for ill-conditioned. Furthermore, polar coordinate form prevents switching all of PV type buses to PQ that happened in rectangular form [17, 18, 19]. The contribution of this paper is apparent in introducing the proposed algorithm based on OMLFM in order to obtain LVS's and MLP simultaneously. To evaluate the proposed method, case studies based on IEEE 57 bus test system and and 13 bus radial transmission system, in well and ill-conditioned systems are tested [20].

- *A. Shahriari is with Department of Electrical Engineering, Faculty of Engineering, University of Malaya, Kuala Lumpur 50603, Malaysia. PH-+60129723448. E-mail: shahriariamid@siswa.um.edu.my*
- *H. Mokhlis is with Department of Electrical Engineering, Faculty of Engineering, University of Malaya, Kuala Lumpur 50603, Malaysia. E-mail: hazli@um.edu.my*
- *Ab. Halim Abu Bakar is with UM Power Energy Dedicated Advanced Centre (UMPEDAC), Level 4, Wisma R&D, University of Malaya, Malaysia. E-mail: a.halim@um.edu.my*
- *M. Karimi is with Department of Electrical Engineering, Faculty of Engineering, University of Malaya, Kuala Lumpur 50603, Malaysia. E-mail: mazaher@ siswa.um.edu.my*
- *M. M. Aman is with Department of Electrical Engineering, Faculty of Engineering, University of Malaya, Kuala Lumpur 50603, Malaysia.*

2 The Improved Optimal Multiplier Load Flow Method

The power flow equations as a set of nonlinear equations can be written as:

$$F(X, H) = 0 \quad (1)$$

Where
X - Vector of independent (state) variables.
H - Vector of dependent variables

F- Load flow function

The E.q (1) involves only two variables i.e. bus voltage amplitude (X_a) and its angle (X_b) at buses as independent variables in polar coordinate form. Since, Taylor series, expansion of E.q (1) based on these state variables is formulated as follow:

$$F(X_a^{\,i}+\Delta X_a^{\,i}, X_b^{\,i}+\Delta X_b^{\,i}) - F(X_a^{\,i}, X_b^{\,i}) - \left[\Delta_{X_a,X_b} F\right]^i \left[\Delta X_a^{\,i}, \Delta X_b^{\,i}\right]$$
$$-\frac{1}{2}\left[\Delta X_a^{\,i}, \Delta X_b^{\,i}\right]^T \left[\Delta^2{}_{X_a,X_b} F\right]^i \left[\Delta X_a^{\,i}, \Delta X_b^{\,i}\right] = 0 \qquad (2)$$

The standard Newton-based methods can solve E.q (2) by neglecting the second term E.q (2). But in ill conditioned system first initial estimation of newton-based methods state variables such as Newton Raphson Load Flow Method (NRLFM) is far away from real solution. For this purpose, the Optimal Multiplier Load Flow Method (OMLFM) modifies the direction of state variables in state space from first initial estimation in order to find best stable solution or low voltages solutions [7, 13]. Therefore, the modification of next step of state variables is

$$X^{i+1} = X^i + \lambda \Delta X^i \qquad (3)$$

where λ is multiplier damper that is used to modify the mismatch vector of state variables. Since, E.q (3) can be rewritten by using λ

$$F(X_a^{\,i}+\Delta X_a^{\,i}, X_b^{\,i}+\Delta X_b^{\,i}) - F(X_a^{\,i}, X_b^{\,i}) - \lambda^i \left[\Delta_{X_a,X_b} F\right]^i \left[\Delta X_a^{\,i}, \Delta X_b^{\,i}\right]$$
$$-\frac{1}{2}\lambda_i^2 \left[\Delta X_a^{\,i}, \Delta X_b^{\,i}\right]^T \left[\Delta^2{}_{X_a,X_b} F\right]^i \left[\Delta X_a^{\,i}, \Delta X_b^{\,i}\right] = 0 \qquad (4)$$

λ is called optimal multiplier due to optimization method based on nonlinear programming technique is utilized to calculate it. Under this condition the multiplier cost function in E.q (5) is defined as objective function in order to obtain then the correction value of state variable as follows

$$C = \frac{1}{2}\Big|\ F(X_a^{\,i}+\Delta X_a^{\,i}, X_b^{\,i}+\Delta X_b^{\,i}) - F(X_a^{\,i}, X_b^{\,i}) - \lambda^i \left[\Delta_{X_a,X_b} F\right]^i \left[\Delta X_a^{\,i}, \Delta X_b^{\,i}\right]$$
$$-\frac{1}{2}\lambda_i^2 \left[\Delta X_a^{\,i}, \Delta X_b^{\,i}\right]^T \left[\Delta^2{}_{X_a,X_b} F\right]^i \left[\Delta X_a^{\,i}, \Delta X_b^{\,i}\right] = 0 \qquad (5)$$

We can get by manipulating (5) to

$$= \frac{1}{2}[D^T D + 2D^T E\lambda + (E^T E + 2G^T D)\lambda^2 + 2E^T G\lambda^3 + GG^T\lambda^4] \qquad (6)$$
$$C = \frac{1}{2}\left[D + E\lambda + \lambda^2 G\right]^T \left[D + E\lambda + \lambda^2 G\right] \rightarrow$$

where

$$D = F(X_a^{\,i}+\Delta X_a^{\,i}, X_b^{\,i}+\Delta X_b^{\,i}) - F(X_a^{\,i}, X_b^{\,i})$$
$$B = \left[\Delta_{X_a,X_b} F\right]^T \left[\Delta X_a^{\,i}, \Delta X_b^{\,i}\right]$$
$$C = \frac{1}{2}\lambda_i^2 \left[\Delta X_a^i, \Delta X_b^i\right]^T \left[\Delta^2 X_a, X_b F\right]^T \left[\Delta X_a^i, \Delta X_b^i\right]$$

Iba et al. improved that the optimum points of multiplier cost function as quartic function in E.q (6) has a two minimum points and one maximum point [14]. The assumption illustration of E.q (6) is depicted in Fig.1. In well conditioned system, the classical newton-based method can only detect on these minimum points as interested solution point called the high voltage solution (HVS). For this case λ is equal to one. In ill conditioned system, by increasing the load demand of system, two minimum points of E.q (6) are forced to get closer to each other and meet at MLP located at maximum point of Fig.1. For this condition the closet value of λ to one is selected as the optimal multiplier to find LVS. Furthermore, it can be observed that E.q (6) is based on NRLFM.

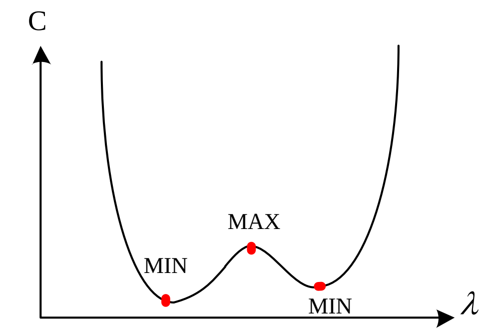

Fig.1.Illustration of assumption multiplier cost function respect λ.

However, optimal multiplier λ is computed by minimizing E.q (6) respect with λ as

(7)

$$\frac{dL}{d\lambda} = D^T E\lambda + (D^T D + 2G^T D)\lambda + 3E^T G\lambda^2 + 2GG^T\lambda^3 0$$

Where *E*=-*D* in order to simplify (7) [13].

The Cardan method is used to find possible real roots of this cubic equation. E.q (7) has three real roots in well-conditioned system. The numbers of real solutions of E.q (7) are decreased at MLP. E.q (7) does not have any real solution in infeasible operation zone beyond MLP.

The proposed algorithm based on OMLFM to calculate LVS in terms of load demand increasing is as follows

Increase predefined load demands based defined step then run NRLFM

1-If NRLFM converges go to 1 otherwise go to 3

2-Calculate the closet optimal multiplier to one of equation (6) by using cardan method due to calculate low voltage solutions.

3-Increase load demands based defined step

4- If that E.q (7) has a real solution (as index to approaching to MLP) stop process and calculate the LVS based on this optimal multiplier otherwise go to (3).

ISSN 2229-5518

3 CASE STUDY

3.1 IEEE 57- bus test system

IEEE 57- bus test system is tested to show convergence characteristics of OMLFM at MLP as ill-conditioned system. The linear active and reactive load demand models are utilized to detect the maximum loading point of this system as follows [15]:

$$P = P_0 + \alpha P \quad (8)$$

$$Q = Q_0 + \alpha Q \quad (9)$$

Where P_0 and Q_0 are the intial and actual vector of active and reactive powers. Alfa (α) involves the step size and the direction the load demand changing. The calculated MLP based on proposed algorithm with α=0.001 this system is 1.92. The newton-based methods cannot converge in ill conditioned system due to the fact the region of LVS if far away from their initial guess. The Table 1 shows the divergence characteristics of NRLFM in calculating LVS's at MLP of IEEE 57 bus. Also, the drastic difference between load flow solutions in ill and well -conditioned for both systems confirms that the result of NRLFM is not at vicinity of its initial guess in ill conditioned system.

TABLE 1
The Performance of Standard Newton Raphson Method for IEEE 57 Bus System In Well and Ill Conditioned System

Bus Number	*Well-conditioned*		*Ill conditioned (MLP)*	
	Voltage (P.U)	*Angle (radian)*	*Voltage (P.U)*	*Angle (radian)*
1	1.04	0	1.04	0
2	1.01	-0.0207737	1.01	-26.1035
3	0.985	-0.104665	0.985	-105.865
4	0.98062	-0.128306	0.865587	-125.369
5	0.97649	-0.149555	0.887585	-156.991
6	0.98	-0.151854	0.98	-169.776
7	0.98392	-0.133294	0.890465	-183.943
8	1.005	-0.0785679	1.005	-187.393
9	0.98	-0.167504	0.98	-198.237
10	0.98531	-0.200622	0.806287	-200.476
11	0.97395	-0.177884	0.509543	-188.939
12	1.015	-0.182713	1.015	-194.222
13	0.97937	-0.170652	0.382618	-174.359
14	0.97012	-0.162803	0.102327	-124.705
15	0.98783	-0.12548	0.377095	-93.8269
16	1.01337	-0.154586	0.356664	-155.411
17	1.01746	-0.0941597	0.231617	-73.4889
18	0.99962	-0.209374	0.688326	-143.827
19	0.96778	-0.235657	0.254942	-157.344
20	0.9606	-0.239537	0.0791747	-152.939
21	1.00399	-0.231896	-0.161233	-248.397
22	1.00503	-0.230947	-0.082112	-594.243
23	1.00366	-0.232048	-0.0409509	-608.772
24	0.99534	-0.237434	0.00594188	-389.213
25	0.97804	-0.324235	0.136447	-1171.4
26	0.95515	-0.232163	0.0127438	-64.5743
27	0.97935	-0.205333	0.451047	-197.135
28	0.99513	-0.186871	0.658432	-194.071
29	1.00906	-0.17423	0.80046	-193.041
30	0.95785	-0.334146	0.33876	-1163.4
31	0.93046	-0.346596	0.582395	-810.355
32	0.94372	-0.332325	-0.419363	-694.734
33	0.94143	-0.333027	-0.470902	-610.467
34	0.95513	-0.252822	0.410462	-1870.77
35	0.96213	-0.248674	0.145749	-1885.93
36	0.97178	-0.243984	0.00944599	-1959.29
37	0.98061	-0.240741	-4.66e-005	-1035.23
38	1.00797	-0.228583	-0.0472536	-800.697
39	0.97861	-0.241493	-0.01187	-990.499
40	0.96897	-0.244343	0.00113648	-296.816
41	0.99345	-0.251172	0.265859	433.549
42	0.96251	-0.277073	-0.164078	141.587
43	1.00824	-0.200977	0.00501284	-532.708
44	1.01295	-0.212772	0.0206411	-388.984
45	1.03439	-0.166673	0.186937	-113.135
46	1.05481	-0.200824	0.124945	-274.015
47	1.02788	-0.225154	0.0138056	-1033.15
48	1.02189	-0.22676	0.128666	-634.705
49	1.02861	-0.232764	0.364332	738.626
50	1.01733	-0.24034	-0.149343	-409.312
51	1.04937	-0.223428	0.699269	-209.916
52	0.97902	-0.204253	0.744088	-199.063
53	0.96952	-0.217373	0.731985	-201.888
54	0.99483	-0.207676	0.849921	-202.337
55	1.02927	-0.191602	0.984154	-201.591
56	0.96336	-0.286765	-0.226818	74.5807
57	0.95941	-0.296219	-0.391522	269.576

On the other hand, the value of phase angle at MLP in Table 1 implies the enhancement of buses voltage angle during demand increasing declines the voltage stability margin. Due to voltage stability can be verified as a static issue if the power systems parameters change slowly.

The smallest bus voltage value from the SNRLF is at bus 33 (-0.470902 p.u.). This unreasonable bus voltage value does not have any physical interpretation from power system point of view. The performance of OMLFM to find LVS at MLP is shown in Table 2. On account of value of voltage in TABLE II can be assessed that OMLFM computes the possible lowest LVS. Base on this fact, bus 31 in IEEE 57 bus system is the weakest buses in ill conditioned. Other meaning, bus 31 in IEEE 57bus system is selected as the most sensitive bus for contingency analysis for voltage collapse in ill conditioned.

Multiplier cost functions in E.q (6) for 57 IEEE bus system is depicted in Fig.2. IEEE 57 bus system has the one optimal multiplier at MLP. The value of optimal multiplier is 7.50337e-005 shown by X in Fig.2. Indeed, the values of multiplier at minimum points of IEEE 57 bus system multiplier cost function is

ISSN 2229-5518

the optimal multiplier. It can be seen the existence of a single optimal multiplies at MLP. The important point in this context is that the value of optimal multiplier force to zero at MLP. The used scale for multiplier cost function of 57 IEEE bus system is 0.0001.

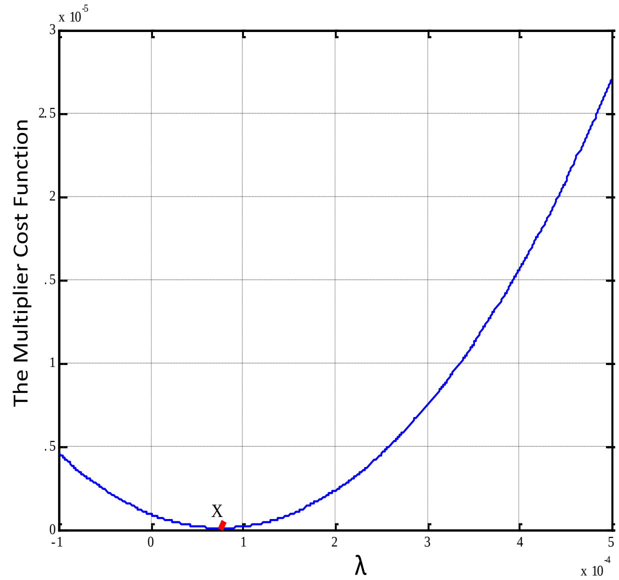

Fig.2. Illustration of multiplier cost function for IEEE 57 bus system.

TABLE2
The Performance of Otimal Multiplier Load Flow Method for IEEE 57 Bus System at Maximum Loading as Point Ill Conditioned System

Bus Number	Well-conditioned Voltage (P.U)	Angle (radian)
1	1.04	0
2	1.01	-8.80125
3	0.985	-36.7282
4	0.957693	-44.3506
5	0.957533	-55.0042
6	0.98	-59.1696
7	0.953584	-62.4881
8	1.005	-60.8325
9	0.98	-64.3329
10	0.942797	-63.8086
11	0.903947	-60.0127
12	1.015	-59.5059
13	0.888769	-54.163
14	0.843531	-48.7822
15	0.878971	-37.6872
16	0.899227	-46.6748
17	0.881131	-25.1099
18	0.91733	-54.8764
19	0.799306	-59.3263
20	0.75621	-60.516
21	0.768149	-61.026
22	0.764134	-61.0184
23	0.758611	-61.4452
24	0.707427	-66.6068
25	0.492606	-83.097
26	0.69687	-66.2363
27	0.826947	-68.045
28	0.890405	-67.4078
29	0.937895	-66.9137
30	0.417639	-84.9749
31	0.318194	-86.862
32	0.392254	-83.2505
33	0.383285	-83.3983
34	0.590846	-65.314
35	0.62457	-64.5221
36	0.662084	-63.6757
37	0.69082	-62.9047
38	0.773927	-60.1987
39	0.686863	-63.1301
40	0.658658	-63.9932
41	0.816818	-70.447
42	0.712185	-73.5239
43	0.901414	-63.0639
44	0.796946	-56.1042
45	0.887031	-46.2266
46	0.882447	-54.8827
47	0.821416	-59.2677
48	0.80816	-59.8521
49	0.835601	-62.307
50	0.839659	-65.6047
51	0.967885	-66.412
52	0.869496	-69.9279
53	0.847242	-71.2201
54	0.918444	-69.638
55	1.00828	-67.4407
56	0.689123	-73.8186
57	0.663633	-74.8809

3.2 13 bus radial transmission system

The 13 bus radial transmission system is considered to verify OMLFM for ill conditioned system includes high R/X ratio lines and the weak interconnection.The 13 bus ill- conditioned systemsis shown in Fig. 3. The lines and buses data is available in [3]. The heavy buses loading, the position of the slack-generator, certain radial system type and two series capacitors result this system to be ill.

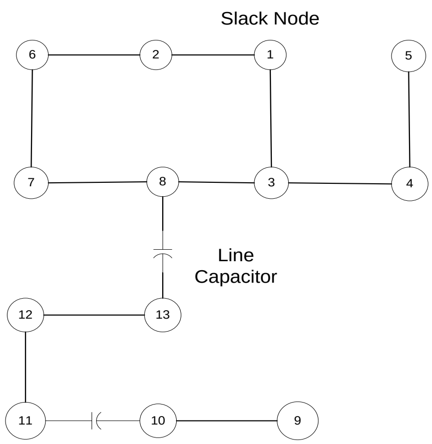

Fig3. The line diagram of 13 bus ill-conditioned system

Under this condition, the load flow equation jacobian becomes singular. Therefore, the eigenvalues of the studied ill-conditioned system's jocobian matrix are very sensitive to small changing in states variables. Fig.4 depicts the sparisity of this singular jacobian matrix. 4. Furthermore, solid geometry of sparse jocobian matrix shown in Fig.5.

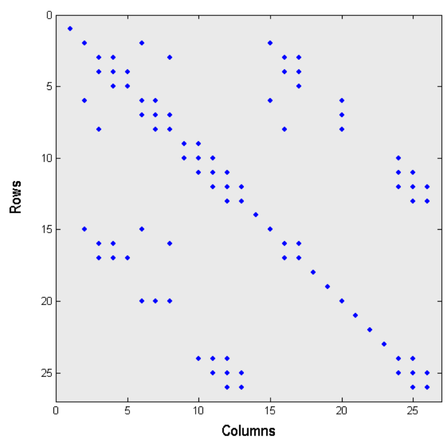

Fig.4. Sparse jacobian matrix for 13bus ill- conditioned system

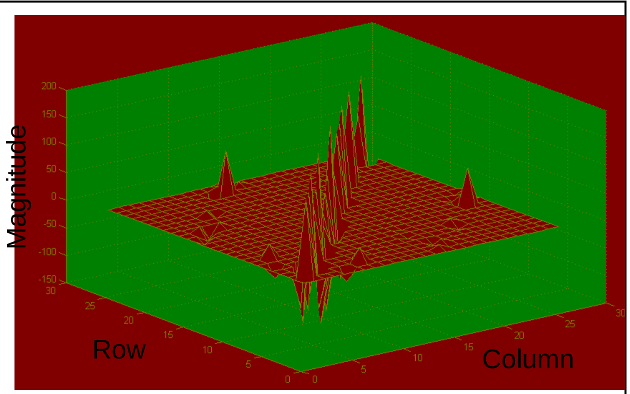

Fig.5. Solid geometry of sparse jocobian matrix of 13 bus ill-conditioned system

For this case, ratio of maximum eigenvalue to minimum eigenvalue as condition the number of the jacobain is very high and is 1000. This is reason of the divergence or osculation of NRLFM in ill conditioned system. OMLFM is used to determine the voltage margin at neighborhood of MLP. thus, OMLFM defines active and reactive power limitations to control buses angles and voltage amplitude. Therefore, bus 13 is used to illustrate the effect of the OMLFM on the improvement of voltage instability of Bus 13 due to its vicinity besides heavy load demanding in Bus 12 and series capacitor between buses in line 13-8. The effect of OMLFM is illustrated in Fig. 6.

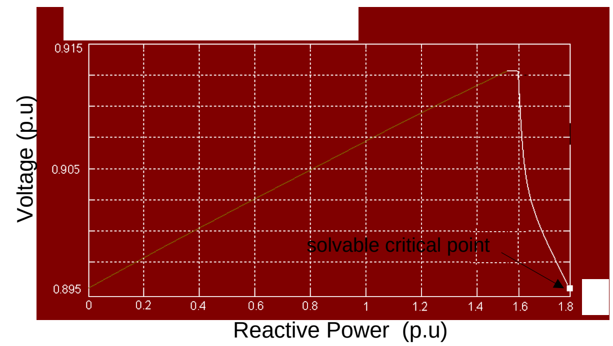

Fig.6. The performance of OMLFM in defining critical point of

Fig.6 shows the enhancement of voltage of bus 13 in term of The increasing of the reactive power demand. It implies the OMLFM attempt to keep voltage amplitude at Bus 13 in acceptable level. It can be seen that the voltage drops significantly at 1.6 p.u of reactive power of bus 13. The MLP of this bus is occurred at 1.8 p.u of its injected reactive power.
The value of voltage of bus 13 at MLP is 0.892 p.u founded by OMLFM. Accordingly, the capability of OMLFM is also apparent in the calculation MLP for ill radial transmission system.

4 Conclusion

This paper presents Optimal Multiplier Load Flow Method (OMLFM) is a robust, reliable and simple method to calculate Low Voltage solution (LVS) in ill conditioned systems. In addition, it has been shown that the proposed algorithms based on OMLFM can obtain LVS's and MLP simultaneously. under this condition , the closet optimal multiplier to one has been selected as the desirable optimal multiplier to find LVS's at MLP. The ability of OMLFM has been validated by IEEE 57 bus test system and 13 bus radial transmission system.

References

[1] B. Stott, "Review of load-flow calculation methods", *Proc. IEEE*, *vol. 62 n. 1*, , pp. 916–929.July 1974.

[2] Wang. Y, da Silva. L.C.P and Wilsun Xu, Investigation of the relationship between ill-conditioned power flow and voltage collapse, *IEEE Power Engineering Review*, pp. 43-45 *vol.20 n.4*, July 2000.

[3] S. C. Tripathy, G. D. Prasad, O. P. Malik, and G. S. Hope, “Load-flow solutions for ill- conditioned power systems by a Newton-like method,” *IEEE Trans. Power App. Syst., vol.101 n.1*, October 1982. pp. 3648–3657

[4] Y. Tamura, H. Mori and S. Iwamoto, “Relationship Between Voltage Instability and Multiple Load Flow Solutions in Electric Power Systems”, *IEEE Trans. Power App .Syst., vol. 102 n. 5* pp.1115 – 1125, May 1983.

[5] R. J. Avalos, C. A. Cañizares, F. Milano, and A. Conejo, Equivalency of Continuation and Optimization Methods to Determine Saddle-node and Limit induced Bifurcations in Power Systems, IEEE Trans. Power Cir. Syst, vol. 56 n.1, January. 2009, pp. 210-223.

[6] Ning Xie, Ettore Bompard, Roberto, Napoli Franco Torelli “Widely convergent method for finding solutions of simultaneous nonlinear equations”. *Electric Power Systems Research journal Volume 83, Issue 1, Pages 1-266* February 2012.

[7] S.K. Goswami, P. Acharjee “Multiple low voltage power flow solutions using hybrid PSO and optimal multiplier method” Expert. Syst. Appl., 37 (2010), pp. 2473–2476

[8] F. Milano, Continuous Newton’s Method for Power Flow analysis, *IEEE Trans. Power Syst, Vol. 24 n.1*, pp. 50-57, February 2009.

[9] J. F. Gutiérrez; M. F. Bedriñana ;C.A.Castro. “Critical comparison of robust load flow methods for ill-conditioned systems” *IEEE Conference Trondheim*, 2011

[10] Shao-Hua Li and Hsiao-Dong Chiang, **“**Continuation Power Flow With Nonlinear Power Injection Variations: A Piecewise Linear Approximation”, *IEEE Trans. Power Syst, vol. 23 n.4* pp. 1637 – 1643., November 2008.

[11] Y. Zhang, H. Chiang, “Fast newton-FGMRES solver for largescalepower flow study,” IEEE Transactions on Power Systems, vol. 25,no. 4, pp. 769–776, May 2010.

[12] G.-Y. Cao and D. Hill, “Power system voltage small-disturbance stabilitystudies based on the power flow equation,” Generation, TransmissionDistribution, IET, vol. 4, no. 7, pp. 873–882, july 2010.

[13] S. Iwamoto and Y. Tamura, A load flow calculation method for ill conditioned power systems, *IEEE Trans. Power App. Syst., vol. 100 n .3*, , pp. 1736–1743,April1981.

[14] K. Iba, H. Suzuki, M. Egawa, and T.Watanabe, A method for finding a pair of multiple load flow solutions in bulk power systems, *IEEE Trans. Power Syst., vol. 5 n.2*, pp. 582–591, May 1990.

[15] T. J. Overbye and R. P. Klump, Effective calculation of power system low -voltage solutions, *IEEE Trans. Power Syst., vol. 11 n.1* pp.75-82 ., February 1996.

[16] M. D. Schaffer and D. J. Tylavsky, A nondiverging polar form Newton-based power flow, *IEEE Trans. Ind. App., vol. 24 n.1*, pp.870–877.September/October 1988.

[17] L. M. C. Braz, C. A. Castro, and C. A. F. Murari , A critical evaluation of step size optimization based load flow methods, *IEEE Trans. Power Syst., vol. 15 n.1*, , pp. 202–207, February 2000

[18] J. E. Tate and T. J. Overbye, A comparison of the optimal multiplier in polar and rectangular coordinates, *IEEE Trans. Power Syst., vol. 20 n 4.* , pp. 1667–1674, November 2005.

[19] Huang,W.-T.;Yao,K.-C.; “New network sensitivity-based approach for real time complex power flow calculation” IET *Generation, Transmission & Distribution* 2011 Vol. 6, Iss. 2, pp. 109– 12. 2012

[20] Available: www.ee.washington.edu/research/pstca.