Numerical Estimation of Inter Line Power Flow Controller Based on Injection Model of Synchronous Voltage Sources

Abstract. This paper presents the estimation model of Inter-Line Power Flow Controller (IPFC) based on injection model of synchronization voltage source. The linear model of voltage source inverter is used to estimate steady-state operation of Inter-Line Power Flow Controller. The proposed model can keep the capability of IPFC with the acceptable accuracy. The active and reactive power flow at receiving end line could be controlled separately in this proposed model. Furthermore, the proposed model has these abilities to decrease the reactive power flow and control of the sending bus voltage at the same time. The validation of the proposed method is shown by using two machine systems to estimate suitable approximation of power flow at receiving end line.

Keywords: Injected active and reactive power, Static Synchronous Series Compensator, Inter-line Power Flow Controller, Voltage source inverter, Numerical estimation.

Introduction

Series compensators are implemented in power system to increase the active power flow at the receiving end line. However, the difficulties of series compensators based on thyristors controller are [1, 2]:
1- Increasing the reactive losses.
2- Sub-Synchronous Resonance (SSR) phenomenon with system inductance.
3- Operation of series compensators depends on line current.
4- Slow operation of series compensators by using the thyristor switch.

Static Synchronous Series Compensator (SSSC) can be utilized to solve the aforesaid problems. For instance, the implementation of SSSC in the power system network avoids the SSR phenomenon[3]. However, SSSC could not control the injected active and reactive power of Voltage Source Inverter (VSI) separately. On the other hands, SSSC has not enough capacity to decrease reactive power losses of a system. Two power compensators were introduced to overcome SSSC defects, the Unified Power Flow Controller (UPFC) and Inter-Line Power Flow Controller (IPFC). However, it is known that the capacity of IPFC in injecting active power to a line is more noticeable than UPFC. The Synchronous Voltage Sources (SVS) including the VSI and Voltage Source Converter (VSC) supplier of IPFC is covered by the active power surplus of other parallel line [4, 5]. For this case, IPFC is used to control the line sending active and reactive power separately. Also, IPFC can control the sending bus voltage simultaneously[6].

IPFC in three modes resistive, capacitive and inductive was used to improve problems of SSSC[7, 8]. IPFC in resistive mode adjusts the sending reactive and active power of main system to 0 and 1 respectively. The maximum injected active power from auxiliary system to the main system occurs in order to increase the sending active power in the main system to the maximum value. Furthermore, the sending reactive and active power flows of the main system in inductive mode are decreased [7-10].

Contribution of the proposed method is apparent in the use of linear model IPFC based on injection mode of VSI to estimate operation of IPFC in resistive, capacitive and inductive modes. The proposed model can support the capability of IPFC with acceptable accuracy in these three modes. The proposed model could separate the control of active and reactive power flow at receiving end line for capacitive, resistive and inductive modes. Another ability of this model is decreasing the reactive power flow with the control of the sending bus voltage together. Thus, the formulation of IPFC of the mentioned modes is robust and simpler than previous approaches. In this context, the comparison of these equations of IPFC with values of simulation indicates the accuracy of the proposed method. The comparison is based on two machine systems. Furthermore, this proposed model of IPFC can also be implemented for power system reliability such as voltage stability and state estimation [3, 5, 11-13].

Basic operation of SSSC in Steady-State condition

Fig.1 shows the steady-state model of SSSC as $V_q$ in single line diagram. The phasor diagram of $V_q$ effects on $V_r$ (sending voltage source) and $V_s$ (receiving end voltage source) system and I (line current) are shown in Fig 2. In capacitive mode of SSSC, the sending active and reactive power to end line increases due to the decrease of line reactance. The operation of SSSC in inductive mode is opposite of capacitive mode. Thus, the active and reactive power flow equations of system using SSSC are given as follows (line resistance is neglected):

$$\pm \vec{V}_q = X_s \vec{I}$$
$$V_r = X S \vec{I}$$

where $\delta = \phi_1 - \phi_2$.

Fig.1. Single line diagram of SSSC
Dividing (5) by (6) and manipulating, (7)

\[
\frac{P}{Q} = \frac{V^2}{X_{\text{eff}}} \sin \delta = \frac{X_q(l \pm X_q/X_L)}{(1 \pm \frac{X_q}{X_L})}
\]

Therefore, SSSC is unable to control reactive power and decreases losses reactive power in the studied system. The reactive power is unbalance between the reactive power load and injected reactive power [6, 8, 14, 15].

Equations (3) and (4) as the injected active and reactive power equations of SSSC can be rewritten based on the effect of SSSC reactance compensation \(X_q\) (both inductive and capacitive modes) on line inductive reactance \(X_L\) as follows,

\[
P = V^2 \sin \delta = \frac{V^2}{X_L} \left(1 \pm \frac{X_q}{X_L}\right)
\]

\[
Q = V^2 \cos \delta = \frac{V^2}{X_L} \left(1 - \frac{X_q}{X_L}\right)
\]

Dividing (5) by (6) then manipulating,

\[
\frac{P}{Q} = \frac{V^2}{X_{\text{eff}}} \sin \delta = \frac{X_q(l \pm X_q/X_L)}{(1 \pm \frac{X_q}{X_L})}
\]

Supposed equation (5) is a function with respect to \(X_q/X_L\). Fig. 5 shows the illustration of (5) with respect to \(X_q/X_L\).

**Proposed Injection Model of IPFC in Steady-State Condition**

Synchronous Voltage Sources (SVS) model of IPFC consists of two back-to-back, series connected with lines as converter and inverters as shown in Fig. 4. System 1 is the main system and system 2 is the auxiliary system, is supposed to balance the needed active and reactive power of VSI in system 1 [7, 19-21].

Therefore, the sending active and reactive power equations of the main system that are associated with the injected active and reactive power of VSI are as follows:

\[
P_s = V_{11} V_{12} \sin \delta_s + P_{c_s}
\]

\[
Q_s = V_{11} V_{12} \cos \delta_s - Q_{c_s}
\]

where

\[
P_{c_s} = \frac{V_{11} V_{12}}{X_1} \sin (\varphi_{c_s} + \varphi_{c_s})
\]

\[
Q_{c_s} = \frac{V_{11} V_{12}}{X_1} \cos (\varphi_{c_s} + \varphi_{c_s})
\]

The significant effect of this model is emerged in equations (8) and (9). These equations can be used for exact linear model of IPFC operation which consists of three modes, resistive, capacitive and inductive in power system where

\[
\hat{V}_{11} = V_{11} \angle \varphi_{c_s}, \quad \hat{V}_{12} = V_{12} \angle \varphi_{c_s}, \quad \hat{V}_{c} = V_{c} \angle \varphi_{c_s} = \hat{V}_{c_{\text{opt}}} + j \hat{V}_{c_{\text{opt}}}
\]

and \(\delta_s = \angle \hat{V}_{11}, \hat{V}_{12}\)

a. **Resistive mode of IPFC**

The purpose of a resistive mode of IPFC is to decrease the sending reactive power flow to 0 p.u. and to increase the real power flow at receiving end line to 1 p.u. This means

\[
Q_{c_s} = 0
\]

\[
I_{1p} = I_{1}
\]

where

\[
I_1 = I_{1p} + j I_{1q}
\]

Therefore, using equations (8), (9) and phasor diagram of IPFC for system1 Fig. 5 gives

\[
\hat{V}_{c_{\text{opt}}} = \left(P_{c_s} - \frac{V_{11} V_{12}}{X_1} \sin \delta_s\right) \frac{X_1}{V_{12}}
\]
and real vector of line current. Thus, with respect to the real vector of the injected IPFC voltage injected complex power by IPFC in system 1 is a function of injected reactive power is zero. Under this condition, the characteristics of IPFC for system 2 can also be considered as SSSC. For (18) it is shown that for system 2 phase angle, phase angle between receiving end voltage in system 1. In Fig.8, it is apparent that for system 2 phase angle, phase angle between receiving end line 2 are 180° because system 2 balances the needed active power for system 1.

Fig.7 shows that line current phasor of system1 ($I_1$) has only real vector and $I_1$ is in the same phase with $V_{22}$ as receiving end voltage in system 1. In Fig.8, it is apparent that for system 2 phase angle, phase angle between $I_2$ & $V_{22}$ is 180° because system 2 balances the needed active power for system 1.

From equations (12),
$$I_1 = I_{1p} + jI_{1q} = \frac{P_{21}}{X_1} = \frac{V_{21} - V_{11} \cos \delta_1}{X_1} = \frac{V_{11} \cos \delta_1 + jV_{11} \sin \delta_1 + jV_{cp} - V_{21}}{X_1}$$

From equations (8), (13), (14) and (15),
$$P_{21} = \frac{V_{cp} V_{11}}{X_1} \sin \delta_1 + \frac{V_{cp} V_{11}}{X_1} \sin (\phi_{21} + \phi_2)$$

In this context, phasor diagram of system 1 in Fig. 6 is simplified as shown in Fig. 7.

The active and reactive power flow equations at receiving end line 2 are
$$P_{r2} = \frac{P_{c2} V_{21}}{X_2} \left( V_{21} - V_{11} \cos \delta_1 \right) - P_{c1}$$
$$Q_{c2} = \frac{V_{c2} V_{21}}{X_2} \cos \delta_2 + \frac{V_{c2} V_{21}}{X_2} \cos (\phi_{22} + \phi_2 + \pi)$$

where
$$\vec{V}_{21} = V_{21} \angle \phi_{21}, \quad \vec{V}_{12} = V_{12} \angle \phi_{12}, \quad \vec{V}_{c2} = V_{c2} \angle \phi_{c2}$$
$$\delta_2 = \vec{V}_{22} \angle \phi_{22}, \quad \vec{V}_{12} = V_{12} \angle \phi_{12}, \quad \vec{V}_{c2} = V_{c2} \angle \phi_{c2}$$

For this case, the characteristics of IPFC for system 2 can also be considered as SSSC. For (18) it is shown that
$$Q_{c2} = \frac{V_{c2} V_{21}}{X_2} \cos \delta_2 - \frac{V_{c2} V_{21}}{X_2} \cos (\phi_{22} + \phi_2 + \pi)$$
b. Capacitive mode of IPFC

Principal privilege of IPFC in this mode emerged to improve the aforesaid problems of SSSC. In this way, it provides simultaneous and independent compensation of needed reactive and active power in system 1 by system 2. It can lead system 1 to the maximum active power transfer ability and control the reactive power flow in system 1. Thus, the maximum injected active power IPFC in system 1 is calculated by the derivative of equation (8) with respect to \( \varphi_{c1} \),

\[
P_{c1} = \frac{V_{c1} V_{c1}}{X} \left( \cos \varphi_{c1} \sin \varphi_{c1} + \cos \varphi_{c1} \sin \varphi_{c1} \right) \rightarrow
\]

\[
dp_{c1} = 0 \rightarrow (\cos \varphi_{c1} \cos \varphi_{c1} - \sin \varphi_{c1} \sin \varphi_{c1}) = 0
\]

(20)

\( \varphi_{c1} = \frac{\pi}{2} - \varphi_{c1} \)

Since the maximum sending active power to end line for system can be calculated by substituting \( \varphi_{c1} = (\pi/2) - \varphi_{c1} \) in (8),

\[
P_{c1} = \frac{V_{c1} V_{c1}}{X} \sin \delta_1 + \frac{V_{c1} V_{c1}}{X} \cos \varphi_{c1}
\]

(21)

Therefore, the capacitive mode can be used to solve the problem of increasing and decreasing reactive and active power in transmission line with high X/R ratio respectively. This is because capacitive mode region of system 1 is between 0 and \( \pi/2 \) to \( \pi \) respectively. The capacitive mode region of system 2 is between \( \pi/2 \) to \( \pi \). Thus, system 2 is formulated as

\[
P_{c2} = \frac{V_{c2} V_{c2}}{X_2} \cos \phi_{c2} - \frac{V_{c2} V_{c2}}{X_2} \sin \phi_{c2}
\]

(22)

\[
Q_{c2} = \frac{V_{c2} V_{c2}}{X_2} \sin \phi_{c2} - \frac{V_{c2} V_{c2}}{X_2} \cos \phi_{c2}
\]

(23)

c. Inductive mode of IPFC

In some conditions, an inductive mode of IPFC is used to approach to the minimum value of the injected active power and decreases the injected reactive in system 1 simultaneously. This indicates that

\[
P_T = P_{c1} + P_{c1} \rightarrow
\]

\[
\text{MIN} (P_T) = \text{MIN} (P_{c1} + P_{c2}) \rightarrow
\]

\[
\text{MIN} (P_T) = (P_{c1} + P_{c2}) \rightarrow -\frac{\pi}{2} \leq \varphi_{c1} < 0
\]

(24)

where \( P_T \) is sending active power to end line.

Therefore equations (8), (9) and (24) can be used to find the sending active and reactive power to end line in system 1. The operation region of IPFC in system 1 is between \( -\pi/2 \) and to \( 0 \). Due to the system 2 that supplies system1 active power demanding, the capacitive region of system 2 tends to between \( -\pi/2 \) to \( \pi \). For system 2,

\[
P_{c2} = \frac{V_{c2} V_{c2}}{X_2} \sin \delta_2 - \frac{V_{c2} V_{c2}}{X_2} \sin \phi_{c2} - \frac{\pi}{2}
\]

(25)

Thus, reactive power flow is similar to capacitive mode.

Case study

The proposed method has been tested on two machine system as same as IPFC in Fig.3[10]. The simulation is based on resistive, capacitive and inductive modes. The buses of two systems have been assumed ideal (infinite bus). The voltage of two system generators are 1 P.U. with a 30° and line reactance of 0.5 P.U.

a. Resistive mode

In this case, the voltage phasor, \( V_{c1} \) with a magnitude of 0.134 P.U is injected at -30°. \([P_{r1} , Q_{r1}] \) is changed from [1.0, -0.2681] P.U to [1.0, 0] P.U. \([P_{r2} , Q_{r2}] \) leads to [0.866, -0.53] P.U from [1.0, -0.2681] P.U. Therefore, equations (8), (9), (17), (18) and (19) are used for numeral estimation. Results of resistive mode are shown in Table 1.

Table 1. Reactive and reactive power flow estimation of system 1 and 2 (p.u) in resistive mode

<table>
<thead>
<tr>
<th>Capacitative \ Mode</th>
<th>Simulation</th>
<th>Proposed method</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr1</td>
<td>1.00019</td>
<td>1.00019</td>
<td>0.02</td>
</tr>
<tr>
<td>Qr1</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.00</td>
</tr>
<tr>
<td>Pr2</td>
<td>0.86619</td>
<td>0.86619</td>
<td>0.21</td>
</tr>
<tr>
<td>Qr2</td>
<td>-0.5356</td>
<td>-0.5356</td>
<td>1.45</td>
</tr>
</tbody>
</table>

b. Capacitive mode

For capacitive mode, a voltage phasor \( V_{c1} \) with a magnitude of 0.26 P.U is injected at +45°. The values of \([P_{r1} , Q_{r1}] \), with an ideal lossless system is changed to [1.5, -0.1341] P.U. The values of \([P_{r2} , Q_{r2}] \) is kept at [0.866, -0.53] P.U. For estimation of this mode, equations (8), (9), (21), (22), (23) are applied. The results are shown in Table 2.

Table 2. Active and reactive power flow estimation of systems 1 and 2 (p.u) in capacitive mode

<table>
<thead>
<tr>
<th>Capacitative \ Mode</th>
<th>Simulation</th>
<th>Proposed method</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr1</td>
<td>1.5219</td>
<td>1.5219</td>
<td>1.35</td>
</tr>
<tr>
<td>Qr1</td>
<td>-0.1338</td>
<td>-0.1338</td>
<td>0.3</td>
</tr>
<tr>
<td>Pr2</td>
<td>0.8654</td>
<td>0.8654</td>
<td>0.069</td>
</tr>
<tr>
<td>Qr2</td>
<td>-0.5276</td>
<td>-0.5276</td>
<td>0.569</td>
</tr>
</tbody>
</table>

c. Inductive mode

A voltage phasor \( V_{c2} \) with a magnitude of 0.26 P.U is injected at -75°. The values of \([P_{r1} , Q_{r1}] \) is kept at [0.634, 0.0981] P.U as the same as the capacitive mode. \([P_{r2} , Q_{r2}] \) remains at [0.866, -0.53] P.U. For estimation of this mode, equations (8), (9), (23), (25) are used for this case and the results are shown in Table 3.

Table 3. Active and reactive power flow estimation of systems 1and 2 (p.u) in inductive mode

<table>
<thead>
<tr>
<th>Inductive \ Mode</th>
<th>Simulation</th>
<th>Proposed method</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr1</td>
<td>0.634</td>
<td>0.632</td>
<td>0.315</td>
</tr>
<tr>
<td>Qr1</td>
<td>0.098</td>
<td>0.099</td>
<td>1.01</td>
</tr>
<tr>
<td>Pr2</td>
<td>0.866</td>
<td>0.8654</td>
<td>0.069</td>
</tr>
<tr>
<td>Qr2</td>
<td>-0.5276</td>
<td>-0.5276</td>
<td>0.569</td>
</tr>
</tbody>
</table>
The simulation of these modes was carried out by power system analysis tools package in MATLAB software and is depicted in Fig.9. Fig.10 shows the VSI injecting complex power of IPFC in three modes for system 1. Also, system1 line current and series injecting voltage of VSI in system 1 have been simulated as shown in Figs. 10 and 11. The operating points of IPFC for two systems can be illustrated in Fig.12.

Conclusions
In this paper, the characteristics of SSSC and IPFC based on new injection model were considered. Firstly, a linear model of voltage source inverter (VSI) of IPFC was improved. Therefore, robust equations of the active and reactive power of IPFC based on new injection model were used. The estimation of the initial injected voltage and its phase angle of IPFC and SSSC for their modelling was done in practical manner. This model was tested in inductive, capacitive and resistive modes of IEEE two machine systems. The comparison of the results with MATLAB simulation shows that the proposed method in different operating point of IPFC is acceptable. In addition, this method can be extended to acquire the approximation of active and reactive power flow of receiving end line equations with transmission line resistance.

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