Modal Analysis of Dielectric Loaded Coaxial Probe Fed Circular Waveguide Radiator

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Dielectric resonator antennas(DRA) have recently been proposed[1],[2] as a simple efficient nonmetalic radiator,especially for microwave and millimeter wave range of frequencies.Various approximate and rigorous methods such as perturbational and variational approaches as well as more elaborate mode matching techniques have been developed for analysing resonant characteristics and field distributions in nonradiating structures and particularly enclosed configurations[3],[4].DRA residing on an infinite plane fed by a monopole has been treated by applying method of moments to coupled integral equations for equivalent surface currents on dielectric-air interfaces.The method uses free space Green's function which is obviously inappropriate for complex geometrics such as the one shown in Fig.1.In this paper an almost rigorous procedure for analysing DRA in cylindrical enclosures is presented.

Outline of the method: In many typical applications the diameter of the waveguide 2b, is chosen so that only dominant mode TE_{11} is propagating. Diffraction at the open end will cause the back propagation of the same mode with usually a small amplitude and excitation of a few higher evanescent modes.Therefore an appropriate starting point for analysis of the field at the proximity of the impressed source(interior problem) is to assume that the dielectric loaded circular waveguide shorted at one end is semi-infinite(Fig.2), except for TE11 mode which may undergo a partial reflection at the open end.Reflection coefficient Γ for a TE11 wave incident onto the open end has already been derived [6].Based upon these assumptions,fields generatted by an infinitesimal z-directed electric current element located at r=(r0,0,z') may now be evaluated. To this end source and fields are expanded in terms of ϕ-harmonics ($\sin p\phi, \cos p\phi$).Thus the analysis given below relates the p-th harmonic of the fields to the same harmonic of source.The fields generated by the p-th harmonic of the point source namely $J_p \cos(p\phi)$ constitute the p-th harmonic of the dyadic Green's function $\overleftrightarrow{G}$ for electric field.If we denote the solenoidal part of G by g then [7]:

$$\overleftrightarrow{G}(\vec{r},\vec{r}\,')=\overleftrightarrow{g}(\vec{r},\vec{r}\,')-\hat{z}\hat{z}\delta(\vec{r},\vec{r}\,')/k^2 \tag{1}$$

to find $\overleftrightarrow{g}$ it is noted that the fields throughout the structure excluding immediate proximity of the source ($|z-z'|\le\epsilon$) may be expanded in terms of eigenmodes of the corresponding transverse section of the waveguide:

$$\left.\begin{aligned}\vec{E}&=\sum_n C_n\cdot\sin\beta_{1n}(z+s+\tfrac{1}{2}l)\cdot\vec{e}_{1n}\\ \vec{H}&=\sum_n jC_n\cdot\cos\beta_{1n}(z+s+\tfrac{1}{2}l)\cdot\vec{h}_{1n}\end{aligned}\right| \quad z<-\tfrac{1}{2}l \tag{2a}$$

$$\left.\begin{aligned}\vec{E}&=\sum_n\{(B_{+n}\cdot\exp[-j\beta_{2n}(z-z')]+B_{-n}\cdot\exp[j\beta_{2n}(z-z')]\}\cdot\vec{e}_{2n}\\ \vec{H}&=\sum_n\{(B_{+n}\cdot\exp[-j\beta_{2n}(z-z')]-B_{-n}\cdot\exp[j\beta_{2n}(z-z')]\}\cdot\vec{h}_{2n}\end{aligned}\right| \quad -\tfrac{1}{2}l<z<z' \tag{2b}$$

$$\vec{E}=\sum_n\{(A_{+n}\cdot\exp[-j\beta_{2n}(z-z')]+A_{-n}\cdot\exp[j\beta_{2n}(z-z')]\}\cdot\vec{e}_{2n}$$
$$\vec{H}=\sum_n\{(A_{+n}\cdot\exp[-j\beta_{2n}(z-z')]-A_{-n}\cdot\exp[j\beta_{2n}(z-z')]\}\cdot\vec{h}_{2n} \qquad \Bigg| \; z'<z<\tfrac{1}{2}l \qquad (2c)$$

$$\vec{E}=\sum_n D_n\cdot\exp(-j\beta_{1n}z)\cdot\vec{e}_{1n}+D_1\cdot r_n\cdot\exp(-j\beta_{11}l)\cdot\exp(j\beta_{11}z)\cdot\vec{e}_{11}$$
$$\vec{H}=\sum_n D_n\cdot\exp(-j\beta_{1n}z)\cdot\vec{h}_{1n}+D_1\cdot r_n\cdot\exp(-j\beta_{11}l)\cdot\exp(j\beta_{11}z)\cdot\vec{h}_{11} \qquad \Bigg| \; z>\tfrac{1}{2}l \qquad (2d)$$

where r=non zero only when (p=1,n=1) in this case: $r=\Gamma_{11}\cdot\exp(j\beta_{11}l)$, $(\vec{e}_n,\vec{h}_n)$ are the transverse electric and magnetic modal fields with radial index "n" and azimuthal number "p", β_{in} is the propagation constant for the same mode, and Γ_{11} is the reflection coefficient for TE_{11}. Figure 2 illustrates the case where the point source is located inside the dielectric. The fields due to a source in air, may be derived in a similar manner. However only the first case is treated here. Continuity of transverse fields at the interfaces $z=\pm\tfrac{1}{2}l$ combined with following orthogonality relations:

$$\iint \vec{e}_{1r}\cdot\vec{e}_{1s}\,dS=\delta_{rs} \qquad (3a)$$

$$\iint \vec{h}_{1r}\cdot\vec{h}_{1s}\,dS=\delta_{rs}/Z_{1s} \qquad (3b)$$

provides us with expressions for C_n and D_n in terms of $B_{\pm m}$ and $A_{\pm m}$ respectively. At the same time an infinite order matrix relationship between B_{+m} and B_{-m} and A_{+m} and A_{-m} can be established. Finally, invoking reciprocity between "true" fields (2b,2c) and m-th propagating mode in inhomogeneously filled part of the waveguide, the relationship between the modes' amplitudes in the close vicinity of the source is derived as:

$$A_{\pm m}-B_{\pm m}=\mp\tfrac{1}{2}Z_{2m}J_p\cdot e_{2zm} \qquad (4)$$

where use has been made of the orthogonality relation [5]:

$$\iint \vec{e}_{2r}\times\vec{h}_{2s}\cdot d\vec{S}=\delta_{rs}/Z_{2r} \qquad (5)$$

and e_{2zm}'s are $\hat{z}$-component of electric field of mp-mode. Substituting (4) in continuity relations mentioned earlier yields:

$$[P]\,a_+ + [Q]\,a_- = J_p\,g\,(z') \qquad [U]\,a_+ + [V]\,a_- = 0 \qquad (6)$$

where the matrices' element are:

$$P_{nm}=(q_{nm}\cos\beta_{1n}s+jZ_{1n}t_{nm}\sin\beta_{1n}s)\cdot\exp(\tfrac{1}{2}j\beta_{2m}l)$$

$$Q_{nm}=(q_{nm}\cos\beta_{1n}s-jZ_{1n}t_{nm}\sin\beta_{1n}s)\cdot\exp(-\tfrac{1}{2}j\beta_{2m}l)$$

$$U_{nm}=[(1-r_m)q_{nm}-Z_{1n}t_{nm}(1+r_m)]\cdot\exp(-\tfrac{1}{2}j\beta_{2m}l)$$

$$V_{nm}=[(1-r_m)q_{nm}+Z_{1n}t_{nm}(1+r_m)]\cdot\exp(\tfrac{1}{2}j\beta_{2m}l)$$

$$q_{nm}=\iint \vec{e}_{1n}\cdot\vec{e}_{2m}\,dS \quad \text{and} \quad t_{nm}=\iint \vec{h}_{1n}\cdot\vec{h}_{2m}\,dS$$

where all the surface integrations are performed over the corresponding waveguide cross section and the elements of column vectors a_+, a_- and g are defined as:

$$a_{\pm m}(z')=A_{\pm m}\exp(\pm j\beta_{2m}z')$$

$$g_n(z')=\sum_m \{Z_{2m}\cdot P_{nm}\cdot\cos\beta_{1n}s\cdot\cos\beta_{2m}(\tfrac{1}{2}l+z')-$$

$$Z_{1n}t_{nm}Z_{2m}\cdot\sin\beta_{1n}s\cdot\sin[\beta_{2m}(\tfrac{1}{2}l+z')]\}\cdot e_{2zm}$$

the infinite linear system of equations (6) should be solved for unknowns $A_{\pm m}$ and then using (4) and continuity relations mentioned before other unknowns B_m, C_m, and D_m will also be determined. To solve (6) the infinite system is truncated by considering only a finite number of "N" modes in each region. Thus the matrices in (6) will have dimension NxN. The truncated system can easily be solved for $A_{\pm m}$:

$$a_+=\{A_{+m}\exp(j\beta_{2m}z')\}=J_p\{[P]-[Q][V]^{-1}[U]\}^{-1}\cdot g$$

$$a_-=\{A_{-m}\exp(-j\beta_{2m}z')\}=-[V]^{-1}[U]\cdot g$$

it is interesting to note that t_{nm}'s and q_{nm}'s can be calculated in closed form [4]. Furthermore, the matrices' elements all their operations are independent of z'. Therefore all the numerical computations are performed once and then a finite analytical representation for Green's function is obtained which can be used to determine the field on the radiating aperture and input impedance for a known source distribution.

Numerical results and conclusion: resonant frequency and radiation pattern of a circular waveguide radiator shown in Fig.1 have been analysed by the method presented here. The primary consideration in this special design has been the bandwidth. Experimental results and theoretical predictions for two cases (ϵ_r =4,12) are show in Fig.3. As it is evident from this comparison, the method has a reasonable accuracy. In both cases, a piecewise sinusoidal distribution for the probe current has been assumed. In general a limited number of modes in both inhomogeneous and homogeneous regions provides us with sufficiently accurate results for many practical cases. More rigorous formulation of input impedance and extension to more complex structures are currently under investigation.

References:

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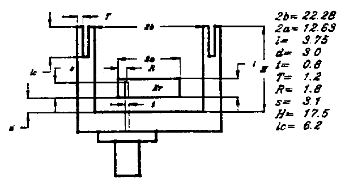

Fig.1 DRA inside a cylindrical enclosure.

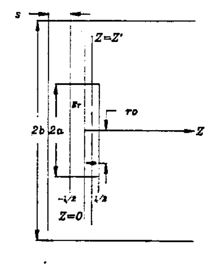

Fig.2 dielectric loaded semini-finite Circular Waveguide

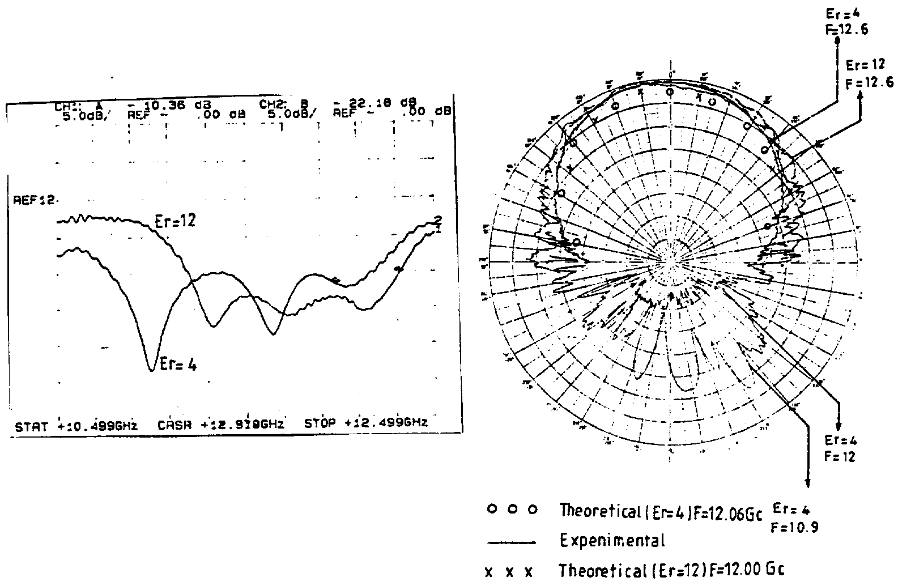

Fig.3 Radiation pattern and return loss.