A HYBRID FINITE-ELEMENT/FINITE-DIFFERENCE SCHEME FOR SOLVING THE 3-D ENERGY EQUATION IN TRANSIENT NONISOTHERMAL FLUID FLOW OVER A STAGGERED TUBE BANK

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This article presents a hybrid finite-element/finite-difference approach. The approach solves the 3-D unsteady energy equation in nonisothermal fluid flow over a staggered tube bank with five tubes in the flow direction. The investigation used Reynolds numbers of 100 and 300, Prandtl number of 0.7, and pitch-to-diameter ratio of 1.5. An equilateral triangle (ET) tube pattern is considered for the staggered tube bank. The proposed hybrid method employs a 2-D Taylor-Galerkin finite-element method, and the energy equation perpendicular to the tube axis is discretized. On the other hand, the finite-difference technique discretizes the derivatives toward the tube axis. Weighting the 3-D, transient, convection-diffusion equation for a cube verifies the numerical results. The L^2 norm of the error between numerical and exact solutions is also presented for three different hybrid meshes. A grid independence study for the energy equation preceded the final mesh. The outcome is found to be in acceptable concurrence with those from the previous studies. After the temperature field is attained, the local Nusselt number is computed for the tubes in the bundle at different times. The isotherms are also obtained at different times until a steady-state solution is reached. The numerical results converge to the exact results through refining the mesh. The implemented hybrid scheme requires less computation time compared with the conventional 3-D finite-element method, requiring less program coding.

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NOMENCLATURE

K	matrix	$\bar{\mathbf{X}}$	coordinate vector
M	mass matrix	θ	dimensionless temperature
P	dimensionless pressure	μ	dynamic viscosity Pa s
$\bar{P}$	Pressure, Pa		
Pe	Peclet number	**Subscripts and Superscripts**	
Pr	Prandtl number	a	advection
Re	Reynolds number	bd	balancing diffusion
t	dimensionless time	d	diffusion
$\bar{t}$	time, s	(e)	element
T	temperature, K	np	total number of nodes in each perpendicular plane
U	velocity, m/s	w	wall
$\bar{\mathbf{V}}$	velocity vector	∞	free-stream
$\mathbf{V} = (u, v, w)$	dimensionless velocity vector		
X	dimensionless coordinate vector		

1. INTRODUCTION

Heat transfer and fluid flow analysis over tube banks is of great importance in various design applications such as cooling towers [1], boilers [2], various heat exchangers [3–5], chemical filtration [6], and nuclear reactors [7]. Heat exchanger design depends on such parameters as the number of pipes of various lengths and diameters, geometric dimensions, arrangement of tubes, and fluids that carry heat [8, 9]. There have been numerous experimental and 2-D numerical studies on the flow over tube banks, but Le Feuvre [10] and later Launder and Massey [11] were the ones who developed numerical applications for fluid flow and heat transfer in tube banks. Alavi and Goshayeshi [12] and Alavi [13] used a finite-element (FE) method to assess the laminar forced convection of air past an in-line and staggered tube bank. Recently, Wu and Che [14] employed a finite-volume (FV) method to examine turbulent forced-convection heat transfer of vapor/air mixture flow past a staggered tube bank. They studied a wide range of tube rows (from 2 to 7), tube diameters, vapor concentration, wall temperature, and gas velocity for an equilateral triangle of tube banks.

However, 2-D analysis is not always sufficient, and a full 3-D analysis is required in some circumstances for more accuracy [15], especially of transfer in heat exchangers [16–18].

There have been a number of 3-D numerical studies on the heat transfer and fluid flow over tube banks. Fan, Ding, He, and Tao [19] developed a code according to nonorthogonal curvilinear coordinates, using the finite-volume method to solve 3-D fluid flow and heat transfer equations around a dimpled fin-and-tube structure. Yusuf, Halvorsen, and Melaaen [20] have studied 3-D turbulent forced-convection heat transfer in a cylindrical pin-fin channel with a staggered and aligned arrangement, using a finite-element commercial code, ANSYS CFX 11. Afgan [15] employed the commercial CFD code STAR-CD to examine the turbulent forced convection of an incompressible Newtonian fluid in a square in-line tube bank. His results showed the 2-D simulations could not capture the complete flow physics. More recently, Iacovides, Launder, Laurence, and West [21] and Iacovides, Launder, and West

[22, 23] examined the turbulent forced-convection heat transfer of a Newtonian fluid around in-line tube banks, using the large-eddy simulation (LES) and unsteady Reynolds-averaged Navier-Stokes (URANS) models in the finite-volume code, Code_Saturne.

Recently, the hybrid scheme has helped overcome the difficulties in generating 3-D meshes. Dai and Nassar [24] studied the heat conduction equation in a double-layered 3-D thin film using a hybrid finite-difference/finite-element method. Their model predictions were in good agreement with earlier finite-difference numerical studies. Chaabane, Askri, and Ben Nasrallah [25] employed the lattice Boltzmann method (LBM) to solve the transient conduction–radiation heat transfer equation in a cylindrical enclosure. In that work, the radiation was solved through a hybrid finite-element/control-volume (CVFEM) approach. Wang, Han, and Sun [26] used a hybrid finite-difference/finite-element scheme to model 1-D and 2-D nonclassical-conduction heat transfer and its related thermal stresses in various configurations. Arefmanesh and Alavi [27] solved the unsteady energy equation in 3-D heat transfer and fluid flow crossing over a circular tube using a hybrid finite-element/finite-difference method.

The aforementioned simulation approaches have proven applicable to 3-D problems. However, complex cases requiring both high computational time and verification of results require a new method. For this article, the hybrid finite-element/finite-difference method, using an in-house FORTRAN code, has numerically analyzed unsteady 3-D, nonisothermal heat transfer and fluid flow past a staggered tube bank. The flow regime's simulation results contrasted with the exact solution results in the literature. Attention focused on computation time and program coding complexity of the proposed method, compared to the 3-D finite-element method.

2. METHODOLOGY

This study solved a 3-D transient energy equation for a nonisothermal fluid flow passing over a staggered tube bank inserted in a channel. To do this, we introduced a hybrid finite-element/finite-difference technique. In this scheme, the finite-difference method discretizes the energy equation along with the tube axis, whereas the finite-element method handles the perpendicular direction. Appointing equally spaced grid points $(m+1)$ along the tube length (H) divides it into identical parts (m). A normal plane with respect to the tube axis is plotted at each point, with Δz the distance between any two sequential planes, as shown in Figure 1 (perpendicular planes). Eventually, symmetry creates an identical 2-D finite-element mesh including triangular elements of three nodes on each of the planes. Thereafter, aligned lines, in parallel with the tube axis, connect the nodes of homological elements in adjacent planes. This creates a hybrid 3-D mesh, as shown in Figure 1.

The following steps create the hybrid finite-element/finite-difference scheme. First, a Navier-Stokes equation solver based on the finite-element method helps calculate the temperature distribution of the flow field. After the velocity distribution is obtained, a 2-D Taylor-Galerkin finite-element scheme helps discretize the energy equation over each perpendicular plane. Finally, finite-difference equations replace resulting derivatives in the obtained semidiscretized equations, which are in the direction of the tube axis (z coordinate).

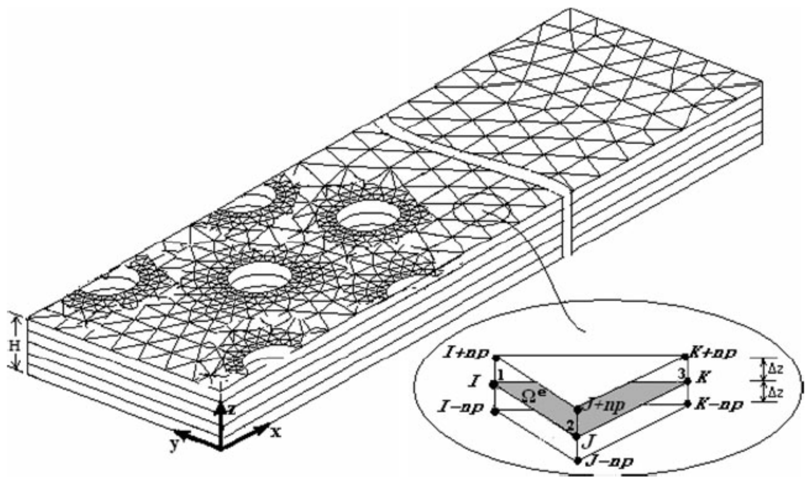


Figure 1. The hybrid 3-D mesh used in the present study.

Dimensionless transient equations of continuity, momentum, and energy for 3-D, incompressible flow of a Newtonian fluid in laminar regime are presented as follows:

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\frac{D\mathbf{V}}{Dt} = -\nabla \mathrm{P} + \frac{1}{\mathrm{Re}}\nabla^2 \mathbf{V} \tag{2}$$

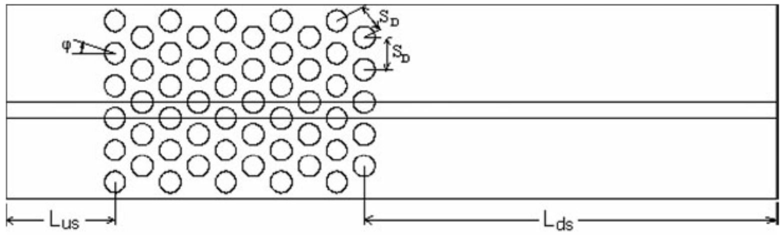
$$\frac{D\theta}{Dt} = \frac{1}{\mathrm{Pe}}\nabla^2 \theta \tag{3}$$

where the dimensionless variables can be expressed as

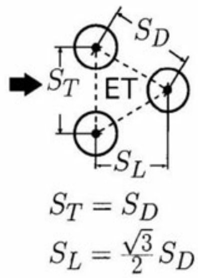
$$\begin{gathered} \mathrm{Pe} = \frac{\rho C_p U_\infty D}{k} \quad \mathrm{Re} = \frac{\rho U_\infty D}{\mu} \quad P = \frac{\bar{\mathbf{P}}}{\rho U_\infty^2} \\ \mathbf{X} = \frac{\bar{\mathbf{X}}}{D} \quad t = \frac{\bar{t} U_\infty}{D} \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \quad \mathbf{V} = \frac{\bar{\mathbf{V}}}{U_\infty} \end{gathered} \tag{4}$$

A physical model of 2-D flow around a staggered tube bank with five tubes in the flow direction is shown in Figure 2*a*. The axis of the first tube is positioned in the upstream at a distance from the input plane equivalent to $3D$ (L_{us}). The computational domain should be long enough on the x axis to fully develop boundary conditions at the outlet plane. Therefore, this length was selected at $12D$, which is up to the fifth tube axis in the downstream (L_{ds}). All the above lengths were chosen according to Tezduyar and Shih [28] and Arefmanesh and Alavi [29]. Various arrangements can be chosen for the tubes in a staggered tube bank. The equilateral triangle (ET) [30, 31] is a tube arrangement used in this work, and its graphical definition is given in Figure 2*b*.

Figure 3 shows the computational domain of the studied problem and its related boundary conditions. The tubes are between two upper and bottom walls



(*a*) Physical model of flow around a staggered tube bank



(*b*) Equilateral Triangle (ET) tube arrangement

Figure 2. Physical model of the analyzed configuration.

at distance *H*. The side walls consist of symmetric planes located between the tubes. No-slip condition is assumed in this simulation, and the tube surface and upper and bottom walls are considered of equal temperature ($T = T_w$). The *x* derivatives of temperature and velocity at the outlet plane are set as zero, because this plane is far from downstream and therefore the flow is considered fully developed. For the symmetry condition, *y* derivatives of all the dependent variables together with the velocity vector in the *y* direction are considered zero along the side walls. At the outlet plane, a zero value is assigned for the pressure. The initial conditions are [29]

$$\mathbf{V}(\mathbf{X}, 0) = 0 \tag{5}$$

$$\theta(\mathbf{X}, 0) = 0 \tag{6}$$

The problem is mathematically modeled by Eqs. (1)–(3) in a coupled system along with the initial conditions in Eqs. (5) and (6) and the boundary conditions in Figure 3. Reynolds numbers of 100 and 300, Prandtl number of 0.7, and a pitch-to-diameter ratio (PDR) of 1.5 are chosen for the investigation.

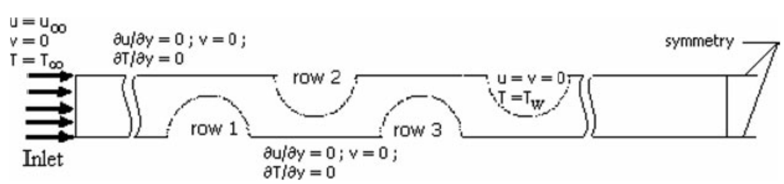


Figure 3. Domain and boundary conditions for the analyzed configuration.

3. NUMERICAL MODELING

The flow field needs to be generated before solving the energy equation. A Navier-Stokes equation solver based on the finite-element technique does this. A linear shape function helps solve the velocity field within tetrahedral elements with four nodes, and the pressure is assumed unchanged in each segment within the elements (piecewise constant pressure) [32]. These approximations are substituted in the Petrov-Galerkin weighted-residual weak form of the Navier-Stokes equations [33] and the time derivatives are discretized. The result is nonlinear algebraic equations in a coupled system for the unknown pressure and velocity components in the nodes [28]. Solution of this system of equations at each time step gives the distributions of velocity and pressure in the computational domain.

The proposed hybrid method is applied to discretize the energy equation [Eq. (3)]. The equation is discretized in the normal planes with respect to the tube axis (x–y planes), as shown in Figure 1. This is done with a 2-D finite-element scheme and 3-noded triangular elements. Thereafter, the finite-difference method discretizes the obtained semidiscretized ODEs in the direction of the tube axis (z direction).

We used the Taylor-Galerkin technique, a proper choice in transient cases, to present the dimensionless form of temperature with respect to time in a second-order-accurate truncated Taylor series in the following equation:

$$\theta^{n+1} = \theta^n + \Delta t \frac{\partial \theta^n}{\partial t} + \frac{\Delta t^2}{2} \frac{\partial^2 \theta^n}{\partial t^2} \tag{7}$$

where n and $n+1$ denote sequential time steps. Using Eq. (3), the tem-perature derivatives of first and second orders with respect to time are given as follows:

$$\frac{\partial \theta^n}{\partial t} = -\mathbf{V} \cdot \nabla \theta^n + \frac{1}{\mathrm{Pe}} \nabla^2 \theta^n \quad \frac{\partial^2 \theta^n}{\partial t^2} = -\frac{\partial \mathbf{V}^n}{\partial t} \cdot \nabla \theta^n - (\mathbf{V}^n \cdot \nabla) \frac{\partial \theta^n}{\partial t} + \frac{1}{\mathrm{Pe}} \nabla^2 \left(\frac{\partial \theta^n}{\partial t} \right) \tag{8}$$

Substituting the expressions of Eq. (8) into Eq. (7), and replacing $\partial \theta^n / \partial t$ by $\theta^{n+1} - \theta^n / \Delta t$ in the ensuing equation, we get a time-discretized form of the energy equation:

$$\left(\frac{1}{\Delta t} - \frac{1}{2\,\mathrm{Pe}} \nabla^2 \right) \theta^{n+1} = \left[\frac{1}{\Delta t} + \frac{1}{2\,\mathrm{Pe}} \nabla^2 - \mathbf{V}^{n+1/2} \cdot \nabla + \frac{\Delta t}{2} (\mathbf{V}^n \cdot \nabla)(\mathbf{V}^n \cdot \nabla) \right] \theta^n \tag{9}$$

The Galerkin finite-element method discretizes Eq. (9) is in the x–y planes. To do so, the following linear interpolation is used to estimate the dimensionless temperature in a typical 3-noded triangular element, Ω^e, within the 2-D mesh (Figure 1):

$$\bar{\theta}^{(e)}(x, y, z, t) = \sum_{i=1}^{3} N_i^{(e)}(x, y) \theta_i^{(e)}(z, t) \tag{10}$$

where $\bar{\theta}^{(e)}$ is the linear approximation of the temperature in the dimensionless form, $N_i^{(e)}$ is the usual linear shape functions, and $\theta_i^{(e)}$ is the value of the dimensionless temperature at the nodes, both for $i = 1$–3.

The Galerkin weighted residual formulation of the problem can be achieved by multiplying Eq. (9) by the shape functions and the integral of the obtained formula over the elements and setting the formula to zero:

$$\int_{\Omega^e} \left\{ \begin{array}{c} \left(\frac{1}{\Delta t} - \frac{1}{2\,\mathrm{Pe}}\nabla^2\right)\theta^{n+1} - \\ \left[\begin{array}{c} \frac{1}{\Delta t} + \frac{1}{2\,\mathrm{Pe}}\nabla^2 - \mathbf{V}^{n+1/2}\cdot\nabla \\ + \frac{\Delta t}{2}(\mathbf{V}^n\cdot\nabla)(\mathbf{V}^n\cdot\nabla) \end{array} \right]\theta^n \end{array} \right\} N_j\,dx\,dy = 0 \tag{11}$$

Using Gauss's theorem and substituting the linear approximations for θ^n and θ^{n+1} from Eq. (10) results in the following system of ordinary differential equations ODEs for the typical element:

$$\left(\frac{\mathbf{M}^{(e)}}{\Delta t} + \frac{\mathbf{K}_d^{(e)}}{2\,\mathrm{Pe}}\right)\left\{\theta^{(e)}\right\}^{n+1} - \frac{\mathbf{M}^{(e)}}{2\,\mathrm{Pe}}\frac{d^2\left\{\theta^{(e)}\right\}^{n+1}}{dz^2} = \left(\frac{\mathbf{M}^{(e)}}{\Delta t} - \frac{\mathbf{K}_d^{(e)}}{2\,\mathrm{Pe}} - (\mathbf{K}_a^{(e)} + \mathbf{K}_{bd}^{(e)})\right)\left\{\theta^{(e)}\right\}^n$$
$$+ \left(\frac{1}{2\,\mathrm{Pe}} + \frac{\Delta t}{2}w^2\right)\mathbf{M}^{(e)}\frac{d^2\left\{\theta^{(e)}\right\}^n}{dz^2} - \left(w^{n+1/2}\mathbf{M}^{(e)} - w\,\Delta t\,\mathbf{K}_a^{(e)}\right)\frac{d\left\{\theta^{(e)}\right\}^n}{dz} \tag{12}$$

where $\{\theta^{(e)}\}$, for $i = 1$–3, is the vector of nodal unknowns. Local node numbers are used to express the elements of the matrices in the following form:

$$\begin{aligned}
\mathbf{M}_{ij}^{(e)} &= \int_{\Omega^e} N_i^{(e)} N_j^{(e)}\,dx\,dy \\
\mathbf{K}_{d_{ij}}^{(e)} &= \int_{\Omega^e} (N_{i,x}^{(e)} N_{j,x}^{(e)} + N_{i,y}^{(e)} N_{j,y}^{(e)})\,dx\,dy \\
\mathbf{K}_{a_{ij}}^{(e)} &= \int_{\Omega^e} (u^{n+1/2} N_{j,x}^{(e)} N_i^{(e)} + v^{n+1/2} N_{j,y}^{(e)} N_i^{(e)})\,dx\,dy \qquad \text{for } i,j = 1-3 \\
\mathbf{K}_{bd_{ij}}^{(e)} &= \frac{\Delta t}{2}\int_{\Omega^e} (u^2 N_{i,x}^{(e)} N_{j,x}^{(e)} + v^2 N_{i,y}^{(e)} N_{j,y}^{(e)} + uv N_{i,x}^{(e)} N_{j,y}^{(e)} \\
&\qquad + uv N_{i,y}^{(e)} N_{j,x}^{(e)})\,dx\,dy
\end{aligned} \tag{13}$$

Discretization of the system of ODEs [Eq. (12)] along with the z axis is the next step of solving the energy equation, done with the finite-difference method. The vector of nodal unknowns in each element is presented according to the global node number:

$$\left\{\theta^{(e)}\right\}^{\ell} = \left\{ \begin{array}{c} \theta_I \\ \theta_J \\ \theta_K \end{array} \right\}^{\ell} \quad \text{for } \ell = n, n+1 \tag{14}$$

where I, J, and K denote the global node numbers in each element (Figure 1). These global node numbers can be used to rewrite the difference quotients for the first and second derivatives with regard to the z axis. Therefore, the dimensionless form of the temperature in Eq. (12) can be given as

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