Stability and Performance Investigations of Model Predictive Controlled Active-Front-End (AFE) Rectifiers for Energy Storage Systems

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Abstract

This paper investigates the stability and performance of model predictive controlled active-front-end (AFE) rectifiers for energy storage systems, which has been increasingly applied in power distribution sectors and in renewable energy sources to ensure an uninterruptible power supply. The model predictive control (MPC) algorithm utilizes the discrete behavior of power converters to determine appropriate switching states by defining a cost function. The stability of the MPC algorithm is analyzed with the discrete z-domain response and the nonlinear simulation model. The results confirm that the control method of the active-front-end (AFE) rectifier is stable, and that it operates with an infinite gain margin and a very fast dynamic response. Moreover, the performance of the MPC controlled AFE rectifier is verified with a 3.0 kW experimental system. This shows that the MPC controlled AFE rectifier operates with a unity power factor, an acceptable THD (4.0 %) level for the input current and a very low DC voltage ripple. Finally, an efficiency comparison is performed between the MPC and the VOC-based PWM controllers for AFE rectifiers. This comparison demonstrates the effectiveness of the MPC controller.

Key words: AC-DC power conversion, Active-Front-End (AFE) rectifier, Energy storage system, Model Predictive Control (MPC), Stability analysis

I. INTRODUCTION

Energy storage systems have been employed in utility and transportation applications as well as in renewable energy sources to ensure power reliability, active power control, load leveling and frequency control [1]-[3]. Generally, energy storage systems use static storage devices such as electric double layer capacitors, Lithium batteries, lead-acid batteries and nickel metal-hydride batteries [4]. These static storage devices contain a high power and energy density but require proper operation such as low ripple current and voltage at the a DC side.

The active front end (AFE) rectifier is an essential part of energy storage systems due to its bidirectional-power-flow, grid synchronization and DC power management capabilities [5]. The control algorithm of the AFE rectifier should be highly stable and efficient since it needs to prevent the problems of poor power quality due to high total harmonic distortion (THD), low power factor, ac voltage distortion, and ripples in the DC current and voltage [5]-[8]. Therefore, several control methods have been investigated to improve the efficiency and performance of the AFE rectifier. The classical control of the AFE rectifier is generally based on a voltage-oriented control (VOC) scheme, which decomposes the active and reactive power into stationary α-β coordinates and synchronizes the powers with the rotating d-q reference frames by characterizing the current control loops by using PI controllers [9], [10]. Moreover, a virtual-flux-oriented control has been proposed in [11], which also uses PI controllers. The major limitation of these control schemes is the need for
tuning the PI controllers that further affects the co-ordinate transform accuracy. Furthermore, direct power control (DPC) schemes [12, 13] that have been applied based on the direct torque control (DTC) [14]-[16] principle also use PI controllers. In order to improve the performance of converters, look-up table (LUT) based direct power control (DPC) schemes have been proposed in [17], [18], in which the switching action of the converter is done with a predefined switching state table based on the basis of the active and reactive power characteristics. The look-up table based DPC methods have a variable switching frequency problem, which produces undesirable harmonic spectrums. To overcome this variable switching, a fuzzy-logic based switching state selection criteria has been presented in [19], [20] to avoid the predefined switching table. Although the active and reactive powers are smoothed in the fuzzy-logic based DPC algorithm when compared with the classical DPC, its sampling frequency is high. Therefore, a sliding mode nonlinear control approach has been investigated in [21] for active and reactive power regulation of grid connected DC-AC converters, which is very much dependent on control variables.

The principle feature of the model predictive control (MPC) scheme is to predict the future behavior of the control variables and it has become an attractive control technique for three-phase AFE rectifiers due to its simple and intuitive concept with fast dynamic responses and flexibility [22]-[24]. Fast and powerful microprocessors are available today to implement predictive control algorithms very easily since they require a higher number of calculations when compared with the classical control methods [25]-[29].

Despite the good performance of the MPC algorithm, there remain some limitations. One of the most important is a stability issue. Recently, the stability analysis of MPC controlled power converters, which are modeled as linear systems has been presented in [30]. The Lyapunov stability investigation of the MPC algorithm has been established in [31]. Furthermore, the Loemberger disturbance and its stability have been observed in [32]. So far, close-loop stability, which is one of the most important aspects of MPC controlled AFE rectifiers, has not been considered.

This paper proposes a model predictive control (MPC) algorithm which is applied in an AFE rectifier to improve its efficiency and performance. The efficiency of this MPC controlled AFE rectifier is compared with a VOC-based PWM rectifier to ensure the effectiveness of the proposed MPC algorithm. Moreover, the close-loop stability criterion of this MPC controlled AFE rectifier has been elaborately investigated with the discrete z-domain response (Root locus, Bode plot, Nyquist diagram and Nichols chart) and the nonlinear simulation model. The rest of the paper is organized in the following manner. The system configuration and mathematical modeling of the three-phase AFE rectifier topology are elaborately described in section II. The formulation of the MPC method with a discrete time model, a cost function for the selection of the switching state and a detailed explanation of the control scheme are described in section III. Section IV provides an analyses of the stability criterion of the model predictive controlled AFE rectifier. The performance of a MPC controlled AFE rectifier prototype is verified and compared with a VOC-based PWM controlled rectifier in section V. Finally some conclusions are drawn in section VI.

II. AFE RECTIFIER TOPOLOGY

A. System Configuration

Fig. 1 shows the three-phase active front end (AFE) rectifier topology which consists of six IGBT-Diode $S_1$-$S_6$ switches. The AFE rectifier is connected with a three-phase voltage supply $v$, using the line filter inductances $L_1$ and resistances $R_e$. A DC capacitor $C_0$ is connected across the resistive load to reduce the DC voltage ripple.

B. Mathematical Modelling

By applying Kirchhoff’s voltage law at the ac side of the
rectifier, the per-phase source-side voltage can be presented as in [33]:

\[ v_{sa} = L_s \frac{di_{sa}}{dt} + R_s i_{sa} + v_{ao} - v_{no} \]  \hspace{1cm} (1)

\[ v_{sb} = L_s \frac{di_{sb}}{dt} + R_s i_{sb} + v_{bo} - v_{ao} \]  \hspace{1cm} (2)

\[ v_{sc} = L_s \frac{di_{sc}}{dt} + R_s i_{sc} + v_{co} - v_{ao} \]  \hspace{1cm} (3)

where, \( v_{sa}, v_{sb}, \) and \( v_{sc} \) are the input phase voltages; and \( i_{sa}, i_{sb}, \) and \( i_{sc} \) are the input phase currents of the three-phase voltage supply for the AFE rectifier.

These input phase voltages and currents can be described with a space vector model. Hence, the space vector model of the three-phase supply voltage and current are:

\[ \bar{v}_s = \frac{2}{3} (v_{sa} + \bar{o} v_{sb} + \bar{o}^2 v_{sc}) \]  \hspace{1cm} (4)

and:

\[ \bar{i}_s = \frac{2}{3} (i_{sa} + \bar{o} i_{sb} + \bar{o}^2 i_{sc}) \]  \hspace{1cm} (5)

where, \( \bar{o} = e^{j2\pi / 3} = -\frac{1}{2} + j\sqrt{3}/2 \). From (1) to (5), the three-phase supply voltage vector can be rewritten as follows:

\[ \bar{v}_s = L_s \frac{d\bar{i}_s}{dt} + R_s \bar{i}_s + \frac{2}{3} (v_{ao} + \bar{o} v_{bo} + \bar{o}^2 v_{co}) \]

\[ = \frac{2}{3} (v_{ao} + \bar{o} v_{bo} + \bar{o}^2 v_{co}) \]  \hspace{1cm} (6)

Note that the last term of (6) is zero since \( 1 + \bar{o} + \bar{o}^2 = 0 \), and the voltage vector \( \bar{v}_R \) generated by the AFE rectifier is:

\[ \bar{v}_R = \frac{2}{3} (v_{ao} + \bar{o} v_{bo} + \bar{o}^2 v_{co}) \]  \hspace{1cm} (7)

This voltage vector \( \bar{v}_R \) is determined from the DC-link voltage \( v_{dc} \) and the switching function vector \( \bar{S}_R \) as:

\[ \bar{v}_R = \bar{S}_R \times v_{dc} \]  \hspace{1cm} (8)

The value of the switching function vector \( \bar{S}_R \) depends on the switching states of the AFE rectifier. In order to avoid short circuits, the two switches in each leg of the rectifier should be operated in a complementary mode. Hence, the switching signals \( S_a \), \( S_b \) and \( S_c \) determine the switching states of the AFE rectifier as follows:

\[ S_a = \begin{cases} 1, & S_1 \text{ is on and } S_2 \text{ is off} \\ 0, & S_1 \text{ is off and } S_2 \text{ is on} \end{cases} \]  \hspace{1cm} (9)

\[ S_b = \begin{cases} 1, & S_3 \text{ is on and } S_4 \text{ is off} \\ 0, & S_3 \text{ is off and } S_4 \text{ is on} \end{cases} \]  \hspace{1cm} (10)

\[ S_c = \begin{cases} 1, & S_5 \text{ is on and } S_6 \text{ is off} \\ 0, & S_5 \text{ is off and } S_6 \text{ is on} \end{cases} \]  \hspace{1cm} (11)

Therefore, the switching function vector \( \bar{S}_R \) of the AFE rectifier can be expressed as:

\[ \bar{S}_R = \frac{2}{3} (S_a + \bar{o} S_b + \bar{o}^2 S_c) \]  \hspace{1cm} (12)

The input current dynamics of (6) becomes:

\[ \frac{di_s}{dt} = \frac{1}{L_s} (\bar{v}_s - \bar{v}_R - R_s \bar{i}_s) \]  \hspace{1cm} (13)

**III. FORMULATION OF THE MPC ALGORITHM**

The MPC algorithm utilizes the discrete nature of the switching devices and the finite number of valid switching states of the power converter. In order to select the appropriate switching state to be applied, the converter switch, a selection criterion must be defined with a cost function which measures the error between the reference and predicted values. Then, the state that minimizes the cost function is selected for switching in the next sampling interval.

**A. Discrete Time Model**

It is important to derive a discrete time model for the power converter system because the predictive controller is formulated in the discrete time domain. To estimate the next sampling value of the input current considering the current and voltage measurements at the \( k \)th sample time, a discrete model of the input side should be employed.

For \( kT_s \leq t \leq (k+1)T_s \), with \( T_s \) being the sampling time, the system model derivative \( dc/dt \) can be expressed from the Euler approximation to increase the fast dynamic response as:

\[ \frac{di_s}{dt} \approx \frac{i_s(k+1) - i_s(k)}{T_s} \]  \hspace{1cm} (14)

Using the above approximation, the discrete time model of the predictive input currents for the next \( (k+1) \) sampling instant of the AFE rectifier can be derived as follows:

\[ i_s(k+1) = \frac{1}{L_s} \left( \frac{R_s T_s}{L_s} \int_{t_k}^{t_{k+1}} v_s \, dt + \frac{T_s}{L_s} \bar{v}_R \right) \]  \hspace{1cm} (15)

**B. Cost Function**

The main objective of the MPC algorithm is to minimize the error with a fast dynamic response between the predicted and reference values of the discrete variables. To achieve this objective, an appropriate cost function \( e \) is defined with a measurement of the predictive input error. Hence, the cost function for the active front end rectifier can be expressed with the absolute error between the predictive and reference values of the input current as:

\[ e = \frac{1}{T_s} \left| \bar{i}_s(k+1) - \bar{i}_s(k) \right| \]  \hspace{1cm} (16)
where, $e$ is the cost function for the AFE rectifier. The reference and predicted input currents of the AFE rectifier are:

\[ \tilde{i}_{\text{ref}}(k+1) \text{ and } \tilde{i}_{p}(k+1). \]

C. Control Scheme

Fig. 2 shows the proposed control strategy of the model predictive control algorithm to operate the active front end (AFE) rectifier. The three-phase input current of the rectifier $i_R(k)$ is measured and the future value of this current $i_R(k+1)$ is predicted by using (15) for each of the eight possible switching vectors $\tilde{S}_R$. The future value of the three-phase input supply current $\tilde{i}_l(k+1)$ is compared with the reference current $\tilde{i}_{\text{ref}}(k+1)$ by utilizing the cost function $e$ of (16). The reference current $\tilde{i}_{\text{ref}}(k+1)$ is calculated from the three-phase input supply voltage vector $\tilde{v}_l$ and the resistive load terminal DC voltage $V_{dc}$ by using a PI controller. Finally, the switching states of the AFE rectifier which minimize the cost function, are selected for the next sampling interval.

D. Transfer Function

The transfer function of the MPC controlled AFE rectifier can be derived from its simplified control scheme, which is presented in Fig. 2. This control structure consists of two cascaded control loops. These are an inner input current control loop and an outer DC link voltage control loop.

1) Current Control Loop: Fig. 3(a) shows a simplified block diagram of the inner input current control loop while considering that the DC link voltage is constant. Therefore, the variation of the supply voltage ($\Delta V_{dc}$) is the only disturbance signal. The error voltage $v_{err}$ is referred to as the difference value of the reference and the predicted values of the converter voltage.

There are many delays in the current control loop such as the processing time of the MPC algorithm, the A/D conversion time, and the delay time of the converter, which have to be taken into account for the control design. Generally, all delays are grouped together to form a single first-order delay element with an equivalent time constant ($T_{eq}$). In the literature [34][36], it has been stated that the converter delay with the MPC algorithm is 1.5 times its sampling time at a 200 μs sampling time with a TMS320C31 DSP processor. The choice of the equivalent delay ($T_{eq}$) mostly depends on the sampling time and speed of the real time interfacing processors which are usually synchronized.

The proposed MPC control scheme of Fig. 2 illustrates the relationship between the sampling instants and the MPC switching signals. It is applied to control the three-phase IGBT converter of the laboratory setup. The MPC algorithm produces a switching signal by accurately tracking the reference value which is symmetrical around the sampling instant. This characteristic of the MPC controlled AFE rectifier makes it possible to sample ripple-free currents. In addition, from a signal point of view, it makes low pass filtering without delay possible. The computation time of the control algorithm ($T_c$) must be shorter than half of the sampling time ($T_s$). From a control point of view it is necessary to define the total delay that varies in the range from 0.5 $T_a$ to $T_a$, depending on the actual control signal. The statistical execution delay of the MPC controller is assumed.
to be 10% of the sampling time $T_s$ (in case of a powerful DSP-based system, e.g., Texas Instruments TMS320F240 processor), where $T_s$ is 50 μs. In addition, considering that the A/D conversion delay is 5% of the sampling time $T_s$, the equivalent time constant $T_{eq}$ can be obtained by utilizing the equation of $T_{eq} = T_s \cdot (10\% + 5\%)$, as follows:

$$T_{eq} = 0.85T_s.$$  \hspace{1cm} (17)

The proposed MPC controlled AFE rectifier uses only a RL filter to connect with a three-phase supply voltage. As a result, the control plant becomes a first-order delay element with a transfer function. Hence, the transfer function of the current control loop can be derived with the equivalent time constants and the input RL filter as:

$$G_s(s) = \frac{K_{MPC}}{s^2T_{eq}L + s(R_{eq}L + L) + (R + K_{MPC})} \hspace{1cm} (18)

where $K_{MPC}$ is the constant gain of the model predictive algorithm which can be defined with the amplitude ratio of the measured and reference currents.

2) DC Link Voltage Control Loop

The dynamic model equation at the DC link voltage of the active front end rectifier is:

$$C_{dc} \frac{dV_{dc}}{dt} = i_{dc} - i_{load} = \frac{3v_{ph}}{2V_{dc}} - i_{load}. \hspace{1cm} (19)

The simplified DC link voltage control loop is presented in Fig. 3(b). Thus, the transfer function of this DC link voltage control loop can be obtained from the PI controller, the inner current loop delay element $T_{inner} = T_{eq}$ and the DC link voltage dynamic equation as:

$$G_s(s) = \frac{K_{p}}{s^2(2V_{dc}C_{dc}T_{inner} + s(2V_{dc}C_{dc} + 3V_{ph}K_p)) + 3V_{ph}K_i} \hspace{1cm} (20)

IV. STABILITY ANALYSIS

Although the model predictive control system is composed of inner current and outer voltage control loops, the inner current control loop is responsible for the overall control system stability [9], [37], [38]. Therefore, the stability analysis of the inner current control loop is carried out with two methods. The first one is a discrete z-domain analysis, which is carried-out by assuming a sample delay with the Root locus, Bode plot, Nyquist diagram and Nichols chart. Afterwards, a Matlab/Simulink model is developed to investigate the stability of the MPC controlled AFE rectifier by means of a nonlinear model.

A. Discrete Z-Domain Analysis

The z-domain stability analysis is based on the current closed-loop transfer function. MPC system plant, delay element and iron losses in the inductors. The Root locus is a powerful method for stability analysis and for measuring the transient response of close-loop control systems which is used for observing the effect of loop gain variations. Moreover, this method can plot the roots of any polynomial with real parameters that hamper the system linearity. Fig. 4(a) depicts the root locus of the inner close-loop current control transfer function which is presented in (18). The root locus technique confirms the stability of the model predictive control since the locus path is lying on the left half of the S-plane.

The stability of the close-loop control system is further analyzed with the discrete z-domain frequency response because of the ambiguities of the root locus sketch, nonlinear system stability, and lead compensators for steady-state errors and transient response. The Nyquist criterion can determine the effects of a time delay on the relative stability of a