Recursive construction of output-context fuzzy systems for the condition monitoring of electrical hotspots based on infrared thermography

Md. Manjur Ahmed [a], A.S.N. Huda [b], Nor Ashidi Mat Isa [a,*]

[a] *Imaging and Intelligent Systems Research Team, School of Electrical and Electronic Engineering, Engineering Campus, Universiti Sains Malaysia, 14300 Nibong Tebal, Penang, Malaysia*
[b] *Department of Electrical Engineering, University of Malaya, 50603 Kuala Lumpur, Malaysia*

ARTICLE INFO

Article history:
Received 17 March 2014
Received in revised form
8 September 2014
Accepted 13 November 2014
Available online 24 December 2014

Keywords:
Electrical hotspots
Condition monitoring
Infrared thermography
Recursive construction
Fuzzy system

ABSTRACT

Infrared thermography technology is currently being used in various applications, including fault diagnosis in electrical equipment. Thermal abnormalities are diagnosed by identifying and classifying the hotspot conditions of electrical components. In this article, a new recursively constructed output-context fuzzy system is proposed to characterize the condition of electrical hotspots. An infrared camera is initially used to capture the thermal images of components with hotspots, and intensity features are extracted from each hotspot. The Recursively Constructed Fuzzy System (RCFS) is then applied to automatically realize and formulate the conditions of the thermal abnormalities. On the basis of the priority level, the hotspot conditions are categorized as normal, warning, and critical. From these three categories, the conditions can be further simplified into two categories, namely, defect (warning and critical) and normal. The proposed RCFS realizes the prominent distinctions in the output domain by using a self-organizing method. The termination of the recursive algorithm finds an effective rule base to achieve an accurate representation of the datasets. The proposed system obtains less fuzzy rules with reasonable accuracy. Our survey of 253 detected regions shows that the proposed RCFS produces 92.3% and 80% testing accuracies for classifying conditions into two and three classes, respectively. The thermographic diagnostic evaluation shows that the proposed intelligent system automatically identifies the rationally acceptable limits of hotspot conditions. Therefore, the proposed system is suitable for establishing an intelligent defect analysis system.

1. Introduction

Heat energy is an important factor in electrical equipment for increasing operational reliability. Electrical current passes through a resistive component and generates heat. The thermal energy generated from an electrical component is directly proportional to the square of the current passing through it and its resistance (I^2R loss). Therefore, an increase in resistance results in an increase in heat. Over time, the condition of the electrical components will begin to deteriorate because of various reasons, such as poor or dirty connections, overloading, insulation problems, load imbalances, corrosion, and wiring mistakes (Korendo and Florkowski, 2001). Components show increased resistance and heat generation with increasing deterioration. The increase in heat energy can cause electrical equipment to fail and fires to break out. The faults caused by the abnormal heating effect can be prevented if heat is detected at an early stage by effective screening and if necessary steps are immediately taken.

Infrared thermography (IRT) senses the heat produced in electrical components. The thermal profiles of different electrical components and connectors are captured by using an infrared camera. The thermal profile (i.e., thermogram) consists of a heat picture and a scale of the temperature values of the equipment. The different colors of the temperature scale represent the different temperature zones in the equipment. By using this profile, thermographers analyze the thermal images and classify the condition of hotspots on the basis of the priority level of repairs. The thermographers then provide suggestions for further action. Finally, the components with hotspots are tested and repaired according to the priority level (Huda and Taib, 2013).

Both manual and Automatic Feature Extraction (AFE) methods are currently employed for the intelligent classification of the thermal

* Corresponding author.
E-mail addresses: manjur_39@yahoo.com (Md.M. Ahmed), nhudaeee0450@gmail.com (A.S.N. Huda), ashidi@eng.usm.my (N.A. Mat Isa).

http://dx.doi.org/10.1016/j.engappai.2014.11.010

conditions of electrical equipment on the basis of thermography. Almeida et al. (2009) proposed an intelligent fault diagnosis system based on thermography for lighting arrestors by using 2 types of variables as inputs of a neuro-fuzzy network. Thermographic and identification variables were used to classify the faults, and the results show approximately 90% accuracy. In one study, RGB color scale data and temperature data were used as the input features of Artificial Neural Network (ANN) to detect internal faults (Shafi'i and Hamzah, 2010). The experiment obtained a 99.38% testing accuracy. Smedberg (2006) and Wretman (2006) proposed an intelligent classification system based on ANN to diagnose 3-phase fuses and different forms of connection problems. The 4 input parameters of ANN were used, namely, absolute max temperature, relative max temperature, mean temperature difference compared with the other regions of the image, and histogram distance to the other regions of the image. The test error rate was 9.5% when all 4 feature parameters are used as ANN inputs, and the error rate was 31.2%when only histogram distance is used as input. The dataset comprised 74 infrared images. One of the disadvantages of working with a small dataset is that the reliability of the results can sometimes be questionable.

On the contrary, several studies have been conducted on the basis of the AFE method and intelligent classification system. Nazmul Huda et al. (2012) proposed a semi-automatic system for electrical thermography. This system uses 15 statistical features and Multi-Layer Perceptron (MLP) network to classify thermal conditions as normal, warning, and critical. This system achieved 78.5% accuracy. Another research proposed an intelligent system to detect faults of electrical equipment in ground substations based on Support Vector Machine (SVM) as a classifier and 22 image features of Zernike moments. The diagnosis obtained 68.42% accuracy (Rahmani et al., 2010). In one study, 10 statistical features and MLP network were employed to differentiate between normal and defective conditions. The system achieved 82.40%accuracy (Nazmul Huda and Taib, 2013). In another study, 6 statistical features and MLP network were used to identify the overheated component; the system reached 79.4% accuracy (Huda et al., 2014).

To develop a robust and reliable system, a new Recursively Constructed Fuzzy System (RCFS) is introduced in this study to classify the conditions of hotspots in components. The proposed system employs AFE and a novel intelligent classification system. The gray scale images of infrared thermal images of components are segmented by using a manual thresholding technique. Then, AFE system automatically extracts six intensity features (i.e., maximum, minimum, mean, median, standard deviation, and variance). The RCFS is a self-automated system that automatically detects the conditions of components and classifies the abnormalities of electrical equipment into classes, namely, normal, warning, and critical. In this study, the RCFS is an output-context fuzzy system that recursively constructs the fuzzy rule base by determining the prominent distinction on the output domain. The termination criterion for recursive algorithm is not threshold as presented in Wang et al. (2010), which realizes from previous and present stages of evolving. The algorithm terminates by recognizing the overfitted partition of the system; therefore, an effective rule base is obtained by the proposed RCFS. After termination, a further evolving process will decrease the model performance. Hence, the computational model of the fuzzy system is automatically designed by the RCFS rather than by human experts.

Neural fuzzy systems are hybrid systems that capitalize on the functionalities of fuzzy systems and neural networks (Nauck et al., 1997). The black-box nature of a neural network can be resolved by integrating the interpretability of a fuzzy system into a connectionist structure (Nauck et al., 1997; Tung et al., 2011). Furthermore, introducing the learning capabilities of a neural network into a fuzzy system will enable the system to automatically refine its parameters (Bosque et al., 2014; Tung et al., 2011). The output-constrained cluster approach (Wang et al., 2011) and Semantic Cointention (SC) approach (Mencar et al., 2011) consider the fuzzy c-means (FCM) to partition data. In the output-constrained cluster approach (Wang et al., 2011), the output space is first roughly partitioned by using FCM. Thereafter, the data within each output constraint are further refined on the basis of "separability," which refers to the connectivity of the inputs. Prior knowledge of rough clustering in the output space makes a fuzzy system unintelligent. The results in Wang et al. (2011) and Mencar et al. (2011) are highly subjective and uncertain because prior knowledge (user-defined number of clusters) was used to design these fuzzy systems. The results are subjective in the sense that the user-defined numbers of clusters are applied to the environment. Nonlinear training or testing errors can be observed in the evaluation, and an absence of overfitting/underfitting assessment is present. Therefore, uncertain results (nonlinear training or testing error) are obtained for some clusters. For example, based on Mencar et al. (2011) for the ionosphere dataset, a nonlinear nature of the testing errors is observed while number of cluster increases. For the automobile dataset, Wang et al. (2011) shows that the training error increases and the testing error decreases with increasing number of rules. Nevertheless, inconsistent results between automobile and census datasets were found. The output domain is evenly partitioned similar to the method of Pedrycz and Kwak (2006); therefore, the output domain ignores the local distribution of the input data. An evenly partitioned output domain may also cause underfitting or overfitting, therefore leading to inaccurate performance. The aforementioned limitations of the existing models are considered in the RCFS. The RCFS is a self-organizing process and evolves by considering both the input and output spaces. The evolving process continues until the termination criteria are fulfilled and the RCFS successfully obtains an effective rule base.

The remainder of this paper is organized as follows. Section 2 describes the necessity of IRT for fault diagnosis. Section 3 discusses the methodology of thermographic diagnosis. Section 4 covers the AFE technique. Section 5 elaborates on the RCFS and its algorithms. Section 6 evaluates the expert system for thermographic diagnosis of electrical components. Section 7 concludes.

2. Importance of infrared thermography-based condition monitoring

The infrared camera is a device that displays the surface temperature of an object by detecting the infrared energy radiated from the surface of this object. The IRT technique is an early internal and external fault diagnosis system for electrical components and provides various advantages over conventional thermal condition and fault diagnosis tools (Kregg, 2004). Some of the advantages of the IRT diagnostic system are described as follows.

2.1. Preventive/predictive maintenance

To maintain electrical equipment, two types of approaches (run-to-failure or preventive maintenance) are used. The run-to-failure approach is simple and straight forward. This approach does not involve an outflow of money for maintenance before the eventual failure of the equipment. The approach waits for equipment failure before any action is taken for maintenance. Therefore, this method is more expensive than preventive maintenance. By contrast, a thermography-based diagnosis system allows preventive/predictive maintenance for the early prevention of equipment failure without interrupting running operations, thus saving money. According to historical data in the United States (TBPPM, 2011), the effective use of preventive/predictive maintenance will decrease about 33–50% of the maintenance cost wasted by most manufacturing and production plants.

(I) **Preventive maintenance:** Preventive maintenance refers to electrical equipment maintenance according to the statistical or historical information on operating capacity, failure history, and Mean-Time-To-Failure (MTTF) instead of tracking equipment performance. A preventive maintenance program schedules the repairing and rebuilding activities for electrical equipment. Suppose an electric component operates for 10 months before needing any repairs. By using the preventive technique, the equipment will be removed from service and rebuilt after 10 months of operation. However, if the equipment does not need to be rebuilt after 10 months, labor and material will be wasted. If the equipment fails before 10 months, the problems should be fixed after failure; this approach is usually more expensive than scheduled maintenance. Note that this case is just a random example and is not supported by any type of empirical data.

(II) **Predictive maintenance:** Predictive maintenance refers to the maintenance of electrical equipment on the basis of the direct monitoring of actual operating conditions and the regular collection of data on measurements, efficiency, heat distribution, and other indicators instead of depending on statistical or historical data. A predictive maintenance program schedules all maintenance activities according to factual data and repairs the equipment if necessary before the occurrence of failure (Epperly et al., 1997).

2.2. *Fire prevention*

According to the reports of the Fire and Rescue Department of Malaysia on the causes of fires in buildings (STAT, 2012), approximately 2317 fire-related incidents have occurred between January 2012 and June 2012, thus making the average number of incidents approximately 387 a month. The report says that a total of 1049 incidents were caused by electrical problems. This figure was almost 46% of the total causes of fires in buildings and mainly involved electrical wiring problems (809 cases) and electrical equipment failure (240 cases). Failure of electrical distribution equipment can produce an ignition and fire. One of the causes of ignition is excessive ohmic heating in electrical distribution. The causes of excessive ohmic heating can be classified into gross overloads, excessive thermal insulation, stray currents, ground faults, overvoltage, and poor connections (Coutin et al., 2012). These conditions tend to occur in old buildings with outdated and deteriorating electrical wirings or electrical wirings that are inappropriately amended or insufficient for the electrical load. However, newly constructed buildings are not immune to these conditions (Plumecocq et al., 2011; Babrauskas, 2001).

2.3. *Reduction of energy loss*

The frequent monitoring of the thermal condition of electrical equipment is necessary to reduce the heat loss that occurs due to elevated surface temperatures. The thermal insulation survey of a 460 MW thermal power station in India reveals that approximately 0.426768 million kJ/h of heat loss occurs because of bare surfaces, inadequate/damaged insulation, or open cladding condition in all four units. This figure is equivalent to a coal loss of about 1847 Mt per annum. Further analysis shows that if the thermal condition these faulty insulated areas are monitored, a financial saving of around 59,052 USD per annum will be achieved, thus amounting to a simple payback period of about one month (Garnaik, 2011).

2.4. *Reduced maintenance cost*

The most efficient and cost-effective ways to increase system reliability include identifying faults quickly before a critical condition arises, scheduling follow-up inspections, and repairing and diagnosing faults within an appropriate period. Thermographic inspection allows for the easy identification of potential problems, quantification of potential energy savings, intervention scheduling, and priority setting for preventive and predictive maintenance or for immediate service to minimize the risk of failure and maintenance cost.

2.5. *Avoiding unnecessary repairs*

Thermographic inspection can display the actual defect area in equipment, thus reducing the need to disassemble, rebuild, repair, or replace good components. Hence, maintenance costs are reduced and revenue is increased.

2.6. *Increased production and safety*

The diagnostic system can diagnose faults without interrupting or shutting down the service, thus resulting in increased production. Furthermore, the failure of electrical components can be catastrophic and can injure or even kill employees, maintenance personnel, or the public.

2.7. *Increased life time*

The power rating of the equipment indicates the amount of energy that the equipment can exert without being damaged. Before failure, the equipment is operated at an excessive power level, which resists the electricity flow and generates heat. Therefore, the equipment overheats and operating efficiency is decreased. However, thermography can increase the lifetime and efficiency of equipment via the early detection of heat.

Table 1
Fluke Ti25 camera specifications.

Temperature range	-20 °C to $+350$ °C	Infrared lens type	20 mm $F=0.8$ lens
Accuracy	± 2 °C or 2%	Image frequency	9 Hz refresh rate
Visual camera	640×480 resolution	Focus	Manual
Detector type	160×120 focal plane array (FPA)	Minimum focus distance	Thermal lens: 15 cm (6 in.); visible (light) visual lens: 46 cm (18 in.)
Field of view	$23° \times 17°$	Spatial resolution	2.5 mrad
Spectral range	7.5–14 μm	Thermal sensitivity	≤ 0.1 °C at 30 °C (100 mK)

3. Methodology of thermographic diagnostics

3.1. Image capture

An infrared camera is used to capture the thermal image of the targeted electrical equipment. In this study, a total of 253 hotspots from 139 infrared images were captured from the Main Switch Boards (MSB). The Fluke Ti25 thermal camera with fusion technology was used to capture the images. Table 1 shows the specifications of Fluke Ti25. For capturing the image, the thermal imager orientation is positioned directly toward the target equipment to obtain an accurate measurement. The emissivity value was set to 0.95, as recommended for the thermography of most types of electrical equipment. Note that the ambient temperature around the equipment is between 30 °C and 33 °C during the inspection.

Selecting an appropriate distance between the target equipment and camera influences the reliability of the thermographic inspection of electrical components. If the selected distance is incorrect, small points in the test object will remain undetected.

A thermal image captured from different distances is presented in Fig. 1. The measured maximum temperatures of the same cable for Fig. 1(a), (b), and (c) are 57.3, 59.1, and 59.5 °C from distances of 5, 2, and 1 m, respectively. For varied distances, a small error occurs in the measurement (Neto et al., 2006; Baranski and Polak, 2010). However, accurately detecting the ROI of equipment is a challenge because of the reduced image size with increasing distance. For example, the image at 5 m in Fig. 1(a) shows a reduced ROI width compared with images at 2 and 1 m. Therefore, identifying the spots that need to be recognized is challenging if the images are randomly taken from long distances.

On the contrary, pixel intensity varies with ROI size and the proposed features depend on the pixel intensities of the target component. Therefore, the thermographic inspection should follow a distance criterion by assuming that the ROI can be focused easily. In this research, we consider the ROIs taken from a distance of 0.5–1 m.

3.2. Typical condition monitoring by IRT

The manual monitoring of component conditions is based on the comparative temperature analysis between hot and reference spots (Huda and Taib, 2013). This technology is simply called qualitative ΔT factor analysis. After capturing the thermal images, the hotspot and reference areas are identified visually by analyzing a color map. The hotspot supports the maximum temperature of faulty components, whereas the reference area is the minimum temperature of the same type, load, or the same repeated component of the equipment. Thereafter, the difference between the hotspot and reference spot temperatures is determined as the ΔT factor, which is used as the decision-making parameter for the condition of the overheated component. The ΔT factor can be interpreted directly from RGB data. Several standards for measuring ΔT include those of the International Electrical Testing Association (STD, 2011) and the American Society for Testing and Materials (ASTM) (ASTM, 2012). The technique is widely used for electrical thermography because of its simplicity and minor emissivity influence. However, the main drawback of this technique is that it does not work in a three-phase system because all phases over heat simultaneously; this situation is infrequent in electrical systems. Fig. 2 shows the flowchart of the IRT inspection of a three-phase electrical system.

In our survey of 253 detected regions, 37 hotspots are critical (ΔT (°C) ≥ 15), 63 spots are in the warning condition ($5 < \Delta T$ (°C) < 15), and 153 spots are normal (ΔT (°C) ≤ 5). All these component conditions (i.e., ΔT (°C)) are evaluated manually by using infrared image analysis software. Some examples of the conditions are illustrated in Fig. 3. Regions with high brightness (denoted by the red color) show more defects than regions with low brightness. For instance,

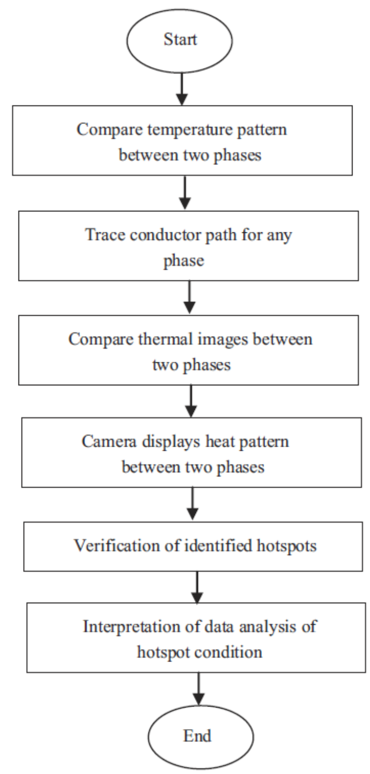

Fig. 2. Flowchart of the infrared thermographic inspection (Bakar et al., 2013).

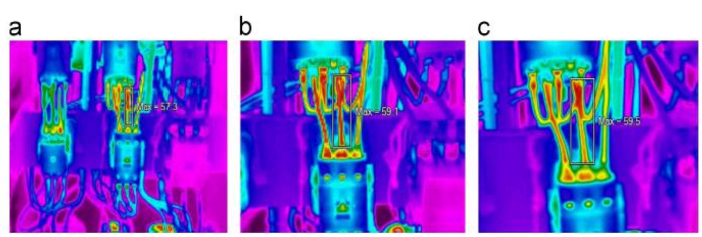

Fig. 1. Thermal images from (a) 5, (b) 2, and (c) 1 m distance.

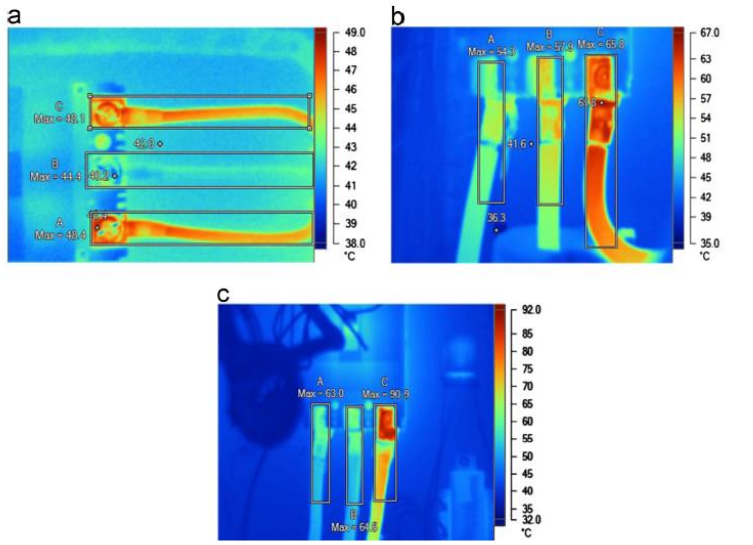

Fig. 3. (a) Hotspots in Phases A and C are normal compared with Phase B, (b) the hotspot in Phase C appears to be in a warning condition compared with that in Phase A, and (c) the hotspot in Phase C appears to be in critical condition compared with that in Phase A. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Fig. 3(c) shows that the maximum temperature values of B and C are 72.7 and 90.9 °C, respectively, whereas 64.6 °C is the maximum temperature value of A or the reference area. The temperature difference between these two phases reveals that B and C are in normal and critical conditions, respectively.

4. Hotspot detection and feature extraction

4.1. Hotspot detection

Daime software version 1.3.1, which is a digital image analysis tool, is used to detect equipment defects and extract quantitative features from defect images (Daims et al., 2006). Segmentation was performed to find the hotspots of electrical components. Daime can segment hot regions on the basis of the threshold value. In the present study, the custom thresholding technique (i.e., manual thresholding) value is set to generate an image of a defect. The image of the defect was produced by setting the threshold value T manually. The original grayscale image shows the hotspot clearly in the value of T; thus, this value was selected as the threshold value for identifying the defect of an image. Thereafter, the image was segmented on the basis of the threshold image.

In this study, two cases were considered to select the desired hotspots: objects with the highest pixel intensity and objects with an area equal to or greater than the half of the maximum area object. The maximum temperature region of an image carries the maximum intensity value. Therefore, the maximum area of the component is generally the region with the highest pixel intensity. In some cases, more than one hotspot is detected in the selected image. Some fake hotspots will be generated because of the electrical installation material and the special structure that causes a high reflection rate or reflection from the sun, thus jeopardizing the infrared emission measurement (Chou and Yao, 2009). By using the object editor, hot defects are manually selected, whereas spurious defects and background are removed. Fig. 4 shows the different steps of defect detection in electrical equipment.

4.2. Automatic feature extraction

A total of six intensity features were computed by using the pixel intensity values of the connected image components. The features are maximum intensity, minimum intensity, average intensity, median intensity, standard deviation, and variance of intensity values. The extracted features are defined as follows:

$$\text{Maximum intensity} = \max \sum_{q=0}^{l-1} q p(q), \tag{1}$$

$$\text{Minimum intensity} = \min \sum_{q=0}^{l-1} q p(q), \tag{2}$$

$$\text{Mean intensity}, \quad \mu = \frac{1}{\sum q} \sum_{q=0}^{l-1} q p(q), \tag{3}$$

$$\text{Variance}, \quad \sigma^2 = \frac{1}{\sum q} \sum_{q=0}^{l-1} (q-\mu)^2 p(q), \tag{4}$$

$$\text{Standard deviation} = \sqrt{\frac{1}{\sum q} \sum_{q=0}^{l-1} (q-\mu)^2 p(q)}, \tag{5}$$

where q is the number of distinct gray levels in the object image, $p(q)$ is the histogram of the object's pixel intensity, and l is the object's possible intensity level. Mean intensity is the average pixel value, which determines the brightness or darkness of the defined object image. Maximum and minimum intensities define the maximum and minimum pixel intensity values of the object image, respectively. Variance determines the dispersion of gray-level pixels from the mean and standard deviation of pixel intensities, which is similar to variance but is different in value. If the intensity values are arranged in ascending order, the middle value is defined as the median intensity value.

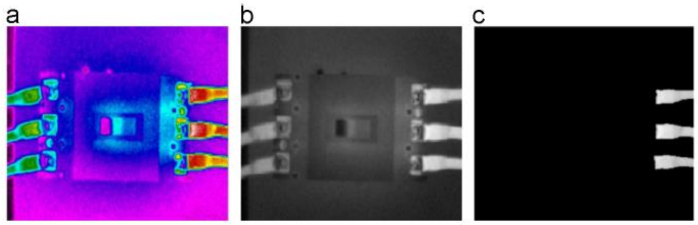

Fig. 4. Typical load imbalance problem: (a) thermal image; (b) grayscale image; and (c) segmented image ($T=163$).

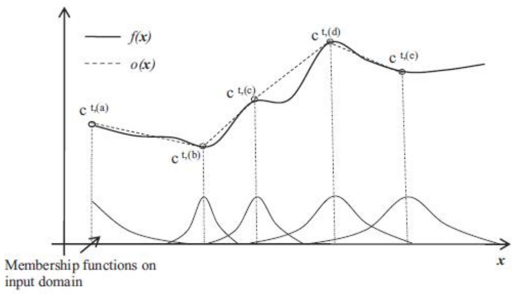

Fig. 5. Realization of prominent distinct points by RCFS. $c^{t,(a)}$, $c^{t,(b)}$, $c^{t,(c)}$, $c^{t,(d)}$, $c^{t,(e)}$ depicts the distinct centers at tth evolving stage.

5. Condition monitoring of hotspots: recursively constructed output-context fuzzy approach

5.1. *Preliminaries: distinct points and effective rulebase*

The main feature of the RCFS is its ability to identify prominent distinct points in the output domain and construct an effective rule base. Fig. 5 illustrates the prominent distinct points in the output domain. A prominent distinction point describes the highly distinct data which depict points with reasonable approximation errors. Therefore, selecting these distinct points to partition the output domain reduces the model error. Wang et al. (2010), Kosko (1995), and Ding et al. (2000) previously described the selection of splitting points to reduce approximation errors. However, unlike the ECSFS (Wang et al., 2010) where the LSM algorithm is used to select the splitting points, the RCFS is evolved and self-determines the distinct output-context to obtain an effective rulebase.

Consider a modeling problem with n input variables and N data samples. Assume an input vector of ith training data $[\boldsymbol{x}, d]_i$, where input vector $\boldsymbol{x}_i$ and the corresponding output d_i and $\boldsymbol{x}=(x_1, x_2, \ldots, x_p, \ldots, x_n)$ belong to an output context (s) that encodes an IF-THEN Mamdani-type fuzzy rule at the tth evolving stage:

$$R^{t,s}: \quad \text{IF } x_1 \text{ is } A_1^{t,(s)}, \text{ and } x_2 \text{ is } A_2^{t,(s)}, \ldots, \text{ and } x_p \text{ is } A_p^{t,(s)} \text{ THEN } y \text{ is } C^{t,(s)}, \tag{6}$$

where $C^{t,(s)}$ is the sth consequent part associated with the sth output context and $A_p^{t,(s)}$ is the sth antecedent part associated with the pth input variable. A Gaussian membership function is described for $C^{t,(s)}$ and its corresponding $A_p^{t,(s)}$.

$$\mu = e^{-((x-c)^2/\sigma)}, \tag{7}$$

and

$$\sigma = \sqrt{-\frac{(\mathrm{a_k}-\mathrm{b_k})^2}{\ln \alpha}}, \tag{8}$$

where c and σ are the center and width of the membership function, respectively; a_k (or b_k) denotes that the data are located at the border of the kth output context; $\alpha>0$is a minimum membership value (Tung and Quek, 2010) that is also a distinguishability factor that maintains the semantic value for the output contexts.

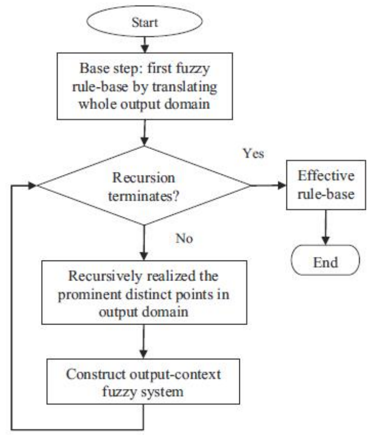

Fig. 6. Flowchart of the RCFS.

The flowchart of the recursive procedure to construct the output-context fuzzy system is shown in Fig. 6. The RCFS employs the Mamdani-type fuzzy system and starts with an initial domain of the whole output space followed by the further partitions of the output domain to identify the prominent distinction point(s). Adaptation is performed for both distinct output context and its corresponding input clusters. In the RCFS, each output-context associated with the input clusters is considered the output-context structure. Previous knowledge in the system and new knowledge from the training data are incorporated in the system to provide an accurate representation of the fuzzy model. Recursive partitioning and evolving processes are continued until a termination criterion is achieved to obtain an effective rulebase.

The output model of the sth domain of the RCFS is defined for the sth rule (or sth output-context) on the basis of Mamdani-type fuzzy systems, which uses the center of the averaging method (Tung et al., 2011; Wang et al., 2011; Tung and Quek, 2010):

$$o = \frac{A_{1,2,\ldots,p,\ldots,n}^{t,(s)}(\boldsymbol{x}) \times c^{t,(s)}|_{\boldsymbol{x} \in domain_s}}{A_{1,2,\ldots,p,\ldots,n}^{t,(s)}(\boldsymbol{x})|_{\boldsymbol{x} \in domain_s}}, \tag{9}$$

where $c^{t,(s)}$ is the center of the sth consequent part of the output context ($C^{t,(s)}$) and

$$A_{1,2,\ldots,p,\ldots,n}^{t,(s)}(\boldsymbol{x}) = \prod_{domain_s} (\mu^{t,(s)}(\boldsymbol{x})) = \prod_{j=1}^{n} (\mu^{t,(s)}(x_j)). \tag{10}$$

The input cluster is adapted to cover all the input data associated with the sth output context. Therefore, Eq. (10) can be represented

Link to Full-Text Articles :

http://www.sciencedirect.com/science/article/pii/S0952197614002826