ESTIMATING THE PERFORMANCE EVALUATION OF INTERNATIONAL AIRPORTS OF IRAN USING KAM IN DATA ENVELOPMENT ANALYSIS

PARVIZ MOHSENPOUR
Iranian Air Transportation Society (IATS), Tehran, Iran

ZAHRA MOHSENPOUR
Iranian Air Transportation Society (IATS), Tehran, Iran

DARIUSH KHEZRIMOTLAGH
Department of Applied Statistics, Faculty of Economics and Administration, University of Malaya, Kuala Lumpur, Malaysia

ABSTRACT

Islamic Republic of Iran has 8 international airports. This paper reports the performance evaluation of these airports in the end of forth development plan of Iran in 2009 by applying Kourosh and Arash Model (KAM) in Data Envelopment Analysis (DEA). The area of airport, apron, terminal and runway are considered as inputs and the number of flights, the number of passengers and cargo are three selected outputs for each airport. Several scenarios are considered to rank and benchmark these airports without concerning about the number of airports. In other words, the scenarios are when inputs/outputs are controllable, when some of them are non-controllable and when the number of flights and passengers are restricted to the set of integer numbers and so on while the number of inputs and outputs are approximately the same as the number of airports. The results of these scenarios not only show the robustness of KAM to assess the performance evaluation of Decision Making Units (DMUs), but they also suggest the best international airports of Iran in 2009 as well as rank and benchmark them for each scenario.

Keywords*: DEA; KAM; Airport; Ranking; Benchmarking.*

INTRODUCTION

Kourosh and Arash Model (KAM) is a recent robust model proposed by Khezrimotlagh *et al*. (2013a, b) in order to improve Data Envelopment Analysis (DEA), proposed by Charnes et al. (1987), in estimating efficiency scores of Decision Making Units (DMUs) as well as their benchmarking and ranking. A review on KAM can be found in Khezrimotlagh et al. (2012a-f, 2013a-e).

In this paper, an applicable study is illustrated on airport efficiency in order to illustrate how KAM is able to measure the efficiency scores of DMUs while the number of DMUs is close to the number of variables. In the first scenario, the data is considered as real data and in the second scenario, the first two outputs are restricted to the set of integer numbers. For the last scenario, the inputs are also considered as non-controllable in order to increase the constraints, and depict robustness of KAM to find the best performers.

AIRPORTS

An international airport is an airport that offers customs and immigration facilities for passengers arriving from other countries. Such airports are usually larger, and often feature longer runways and facilities to accommodate the heavier aircraft commonly used for international and intercontinental travel. International airports often also host domestic flights (flights which occur within the country), to serve travelers to and from these regions of the country. The Islamic republic of Iran has 8 international airports

called OIFM, OIIE, OIII OIKB, OIMM, OISS, OITT and OIZH. The area of airport, apron, terminal and runway are considered as inputs and the number of flights, the number of passengers and cargo are three selected outputs for each airport. There is a good number of studies on airport efficiency which can be seen in (Lozano and Gutierrez, 2011).

DISCUSSION

The number of DMUs in comparison with the number of variables are close in this practice. According to literature of DEA, since the number of variables are 7, the number of DMUs should at least be more than 21, whereas it is only 8 airports. Previous suggestions may say to add data of several years in order to increase the number of DMUs in comparison with the number of variables. However, the inputs of airports are usually constant during the years and such suggestion is not suitable. Moreover, a manager, who has one year experience, would hope to know whether its firm does the job right.

The current methodologies are not able to assess the performance of these airports. For instance, all conventional DEA models identify these airports as technically efficient DMUs in Variable Returns to Scale (VRS) technology (Banker et al., 1985). However, KAM is easily able to distinguish between these airports appropriately, as can be seen in the results section.

On the other hand, the number of passengers and flights are integer and the model should suggest integer targets for benchmarking these variables. Adding such constraints makes the Production Possibility Set (PPS) becomes smaller, and increases the number of technically efficient DMUs. This can be continued when the inputs are considered as non-controllable variables which surely increases the number of technically efficient DMUs. However, KAM in all these situations is able to distinguish between technically efficient DMUs which shows the robustness of KAM in comparison with current non-parametric and parametric technologies.

KOUROSH AND ARASH MODEL

Let us mark OIFM, OIIE, OIII OIKB, OIMM, OISS, OITT and OIZH by DMU_l $(l = 1,2,..,8)$, input variables "the area of airport, apron, terminal and runway" by x_{lj} $(j = 1,2,3,4)$, and output variables "the number of flights, the number of passengers and cargo" by y_{lk} $(k = 1,2,3)$, respectively.

Since the weights of variables are unknown and there is no zero value in data, let us define, $w_{lj}^- = 1/x_{lj}$, $w_{lk}^+ = 1/y_{lk}$, for $j = 1,2,3,4$, $k = 1,2,3$, and $l = 1,2,..,8$.

Assume that $\epsilon_l = (\varepsilon_l^-, \varepsilon_l^+) = (\varepsilon_{l1}^-, \varepsilon_{l2}^-, \varepsilon_{l3}^-, \varepsilon_{l4}^-, \varepsilon_{l1}^+, \varepsilon_{l2}^+, \varepsilon_{l3}^+)$, where $\varepsilon_{lj}^- = \varepsilon x_{lj}$, and $\varepsilon_{lk}^+ = \varepsilon y_{lk}$, for $j = 1,2,3,4$, $k = 1,2,3$, and $l = 1,2,..,8$. Here $\varepsilon \in \mathbb{R}_+$ and it is define as 0.00001 in this paper. The scores of KAM are called with ε-Degree of Freedom (DF) or 0.00001-DF (See Khezrimotlagh, 2014). KAM in VRS is as follows when $\varepsilon = 0.00001$:

$$\max\ s_{l1}^-/x_{l1} + s_{l2}^-/x_{l2} + s_{l3}^-/x_{l3} + s_{l4}^-/x_{l4} + s_{l1}^+/y_{l1} + s_{l2}^+/y_{l2} + s_{l3}^+/y_{l3}, \tag{1}$$

Subject to

$$\lambda_{l1}x_{11} + \lambda_{l2}x_{21} + \lambda_{l3}x_{31} + \lambda_{l4}x_{41} + \lambda_{l5}x_{51} + \lambda_{l6}x_{61} + \lambda_{l7}x_{71} + \lambda_{l8}x_{81} + s_{l1}^- \le x_{l1} + 0.00001 \times x_{l1},$$
$$\lambda_{l1}x_{12} + \lambda_{l2}x_{22} + \lambda_{l3}x_{32} + \lambda_{l4}x_{42} + \lambda_{l5}x_{52} + \lambda_{l6}x_{62} + \lambda_{l7}x_{72} + \lambda_{l8}x_{82} + s_{l2}^- \le x_{l2} + 0.00001 \times x_{l2},$$
$$\lambda_{l1}x_{13} + \lambda_{l2}x_{23} + \lambda_{l3}x_{33} + \lambda_{l4}x_{43} + \lambda_{l5}x_{53} + \lambda_{l6}x_{63} + \lambda_{l7}x_{73} + \lambda_{l8}x_{83} + s_{l3}^- \le x_{l3} + 0.00001 \times x_{l3},$$

$\lambda_{l1}x_{14} + \lambda_{l2}x_{24} + \lambda_{l3}x_{34} + \lambda_{l4}x_{44} + \lambda_{l5}x_{54} + \lambda_{l6}x_{64} + \lambda_{l7}x_{74} + \lambda_{l8}x_{84} + s_{l4}^{-} \leq x_{l4} + 0.00001 \times x_{l4},$
$\lambda_{l1}y_{11} + \lambda_{l2}y_{21} + \lambda_{l3}y_{31} + \lambda_{l4}y_{41} + \lambda_{l5}y_{51} + \lambda_{l6}y_{61} + \lambda_{l7}y_{71} + \lambda_{l8}y_{81} - s_{l1}^{+} \geq y_{l1} - 0.00001 \times y_{l1},$
$\lambda_{l1}y_{12} + \lambda_{l2}y_{22} + \lambda_{l3}y_{32} + \lambda_{l4}y_{42} + \lambda_{l5}y_{52} + \lambda_{l6}y_{62} + \lambda_{l7}y_{72} + \lambda_{l8}y_{82} - s_{l2}^{+} \geq y_{l2} - 0.00001 \times y_{l2},$
$\lambda_{l1}y_{13} + \lambda_{l2}y_{23} + \lambda_{l3}y_{33} + \lambda_{l4}y_{43} + \lambda_{l5}y_{53} + \lambda_{l6}y_{63} + \lambda_{l7}y_{73} + \lambda_{l8}y_{83} - s_{l3}^{+} \geq y_{l3} - 0.00001 \times y_{l3},$
$\lambda_{l1} + \lambda_{l2} + \lambda_{l3} + \lambda_{l4} + \lambda_{l5} + \lambda_{l6} + \lambda_{l7} + \lambda_{l8} = 1,$
$\lambda_{l1} \geq 0, \lambda_{l2} \geq 0, \lambda_{l3} \geq 0, \lambda_{l4} \geq 0, \lambda_{l5} \geq 0, \lambda_{l6} \geq 0, \lambda_{l7} \geq 0, \lambda_{l8} \geq 0,$
$s_{l1}^{-} \geq 0, s_{l2}^{-} \geq 0, s_{l3}^{-} \geq 0, s_{l4}^{-} \geq 0, s_{l1}^{+} \geq 0, s_{l2}^{+} \geq 0, s_{l3}^{+} \geq 0.$

After finding the optimum of slacks, the following targets of Equation (1) are on the real Farrell frontier:

$x_{l1}^{*} = x_{l1} - s_{l1}^{-*} + 0.00001 \times x_{l1},$
$x_{l2}^{*} = x_{l2} - s_{l2}^{-*} + 0.00001 \times x_{l2},$
$x_{l3}^{*} = x_{l3} - s_{l3}^{-*} + 0.00001 \times x_{l3},$
$x_{l4}^{*} = x_{l4} - s_{l4}^{-*} + 0.00001 \times x_{l4},$
$y_{l1}^{*} = y_{l1} + s_{l1}^{+*} - 0.00001 \times y_{l1},$
$y_{l2}^{*} = y_{l2} + s_{l2}^{+*} - 0.00001 \times y_{l2},$
$y_{l3}^{*} = y_{l3} + s_{l3}^{+*} - 0.00001 \times y_{l3}.$

If it is supposed that the first two outputs, the number of passengers and the number of flights, are restricted to the set of integer numbers, KAM is as follows (Khezrimotlagh et al. 2013b, d):

$$\max\ s_{l1}^{-}/x_{l1} + s_{l2}^{-}/x_{l2} + s_{l3}^{-}/x_{l3} + s_{l4}^{-}/x_{l4} + s_{l1}^{+}/y_{l1} + s_{l2}^{+}/y_{l2} + s_{l3}^{+}/y_{l3}, \qquad (2)$$

Subject to

$\lambda_{l1}x_{11} + \lambda_{l2}x_{21} + \lambda_{l3}x_{31} + \lambda_{l4}x_{41} + \lambda_{l5}x_{51} + \lambda_{l6}x_{61} + \lambda_{l7}x_{71} + \lambda_{l8}x_{81} + s_{l1}^{-} \leq x_{l1} + 0.00001 \times x_{l1},$
$\lambda_{l1}x_{12} + \lambda_{l2}x_{22} + \lambda_{l3}x_{32} + \lambda_{l4}x_{42} + \lambda_{l5}x_{52} + \lambda_{l6}x_{62} + \lambda_{l7}x_{72} + \lambda_{l8}x_{82} + s_{l2}^{-} \leq x_{l2} + 0.00001 \times x_{l2},$
$\lambda_{l1}x_{13} + \lambda_{l2}x_{23} + \lambda_{l3}x_{33} + \lambda_{l4}x_{43} + \lambda_{l5}x_{53} + \lambda_{l6}x_{63} + \lambda_{l7}x_{73} + \lambda_{l8}x_{83} + s_{l3}^{-} \leq x_{l3} + 0.00001 \times x_{l3},$
$\lambda_{l1}x_{14} + \lambda_{l2}x_{24} + \lambda_{l3}x_{34} + \lambda_{l4}x_{44} + \lambda_{l5}x_{54} + \lambda_{l6}x_{64} + \lambda_{l7}x_{74} + \lambda_{l8}x_{84} + s_{l4}^{-} \leq x_{l4} + 0.00001 \times x_{l4},$
$\lambda_{l1}y_{11} + \lambda_{l2}y_{21} + \lambda_{l3}y_{31} + \lambda_{l4}y_{41} + \lambda_{l5}y_{51} + \lambda_{l6}y_{61} + \lambda_{l7}y_{71} + \lambda_{l8}y_{81} - s_{l1}^{+} \geq y_{l1} - 0.00001 \times y_{l1},$
$\lambda_{l1}y_{12} + \lambda_{l2}y_{22} + \lambda_{l3}y_{32} + \lambda_{l4}y_{42} + \lambda_{l5}y_{52} + \lambda_{l6}y_{62} + \lambda_{l7}y_{72} + \lambda_{l8}y_{82} - s_{l2}^{+} \geq y_{l2} - 0.00001 \times y_{l2},$
$\lambda_{l1}y_{13} + \lambda_{l2}y_{23} + \lambda_{l3}y_{33} + \lambda_{l4}y_{43} + \lambda_{l5}y_{53} + \lambda_{l6}y_{63} + \lambda_{l7}y_{73} + \lambda_{l8}y_{83} - s_{l3}^{+} \geq y_{l3} - 0.00001 \times y_{l3},$
$\lambda_{l1} + \lambda_{l2} + \lambda_{l3} + \lambda_{l4} + \lambda_{l5} + \lambda_{l6} + \lambda_{l7} + \lambda_{l8} = 1, s_{l1}^{+} \in \mathbb{Z},\ s_{l2}^{+} \in \mathbb{Z},$
$\lambda_{l1} \geq 0, \lambda_{l2} \geq 0, \lambda_{l3} \geq 0, \lambda_{l4} \geq 0, \lambda_{l5} \geq 0, \lambda_{l6} \geq 0, \lambda_{l7} \geq 0, \lambda_{l8} \geq 0,$
$s_{l1}^{-} \geq 0, s_{l2}^{-} \geq 0, s_{l3}^{-} \geq 0, s_{l4}^{-} \geq 0,$
$s_{l1}^{+} \geq 0, s_{l2}^{+} \geq 0, s_{l3}^{+} \geq 0.$

After finding the optimum of slacks for Equation (2), since the two first output constraints are restricted to the set of integer numbers, the target values may not be on the real Farrell frontier, but they are very close to the Farrell frontier with 0.00001-DF, which are given by:

$x_{l1}^{*} = x_{l1} - s_{l1}^{-*} + 0.0001 \times x_{l1},$
$x_{l2}^{*} = x_{l2} - s_{l2}^{-*} + 0.0001 \times x_{l2},$
$x_{l3}^{*} = x_{l3} - s_{l3}^{-*} + 0.0001 \times x_{l3},$
$x_{l4}^{*} = x_{l4} - s_{l4}^{-*} + 0.0001 \times x_{l4},$
$\widehat{y_{l1}^{*}} = y_{l1} + s_{l1}^{+*}, \widehat{y_{l2}^{*}} = y_{l2} + s_{l2}^{+*},$
$y_{l3}^{*} = y_{l3} + s_{l3}^{+*} - 0.0001 \times y_{l3}.$

Note that: If the values of the first two output components of epsilon, that is, $0.00001 \times y_{l1}$ and $0.00001 \times y_{l2}$, are very greater than 1, we have some different alternative decisions according to our goals. For instance we are able to consider the integer values of them, that is, $\lfloor 0.00001 \times y_{l1} \rfloor$ and $\lfloor 0.00001 \times y_{l2} \rfloor$ in Equation (2), and then calculate the targets as $y_{l1}^{*} = y_{l1} + s_{l1}^{+*} - \lfloor 0.00001 \times y_{l1} \rfloor$ and

$y_{l2}^{*} = y_{l2} + s_{l2}^{+*} - \lfloor 0.00001 \times y_{l2} \rfloor$. It is also possible to replace $0.000001 \times y_{l1}$ by $0.00001 \times y_{l1}$ in order to have negligible errors which can be removed clearly.

Now, if it is supposed that the inputs are also non-controllable data, KAM is as follows (Khezrimotlagh et al., 2012c):

$$\max\ s_{l1}^{-}/x_{l1} + s_{l2}^{-}/x_{l2} + s_{l3}^{-}/x_{l3} + s_{l4}^{-}/x_{l4} + s_{l1}^{+}/y_{l1} + s_{l2}^{+}/y_{l2} + s_{l3}^{+}/y_{l3}, \qquad (3)$$

Subject to

$$\begin{aligned}
&\lambda_{l1}x_{11} + \lambda_{l2}x_{21} + \lambda_{l3}x_{31} + \lambda_{l4}x_{41} + \lambda_{l5}x_{51} + \lambda_{l6}x_{61} + \lambda_{l7}x_{71} + \lambda_{l8}x_{81} + s_{l1}^{-} \le x_{l1} + 0.00001 \times x_{l1},\\
&\lambda_{l1}x_{12} + \lambda_{l2}x_{22} + \lambda_{l3}x_{32} + \lambda_{l4}x_{42} + \lambda_{l5}x_{52} + \lambda_{l6}x_{62} + \lambda_{l7}x_{72} + \lambda_{l8}x_{82} + s_{l2}^{-} \le x_{l2} + 0.00001 \times x_{l2},\\
&\lambda_{l1}x_{13} + \lambda_{l2}x_{23} + \lambda_{l3}x_{33} + \lambda_{l4}x_{43} + \lambda_{l5}x_{53} + \lambda_{l6}x_{63} + \lambda_{l7}x_{73} + \lambda_{l8}x_{83} + s_{l3}^{-} \le x_{l3} + 0.00001 \times x_{l3},\\
&\lambda_{l1}x_{14} + \lambda_{l2}x_{24} + \lambda_{l3}x_{34} + \lambda_{l4}x_{44} + \lambda_{l5}x_{54} + \lambda_{l6}x_{64} + \lambda_{l7}x_{74} + \lambda_{l8}x_{84} + s_{l4}^{-} \le x_{l4} + 0.00001 \times x_{l4},\\
&\lambda_{l1}y_{11} + \lambda_{l2}y_{21} + \lambda_{l3}y_{31} + \lambda_{l4}y_{41} + \lambda_{l5}y_{51} + \lambda_{l6}y_{61} + \lambda_{l7}y_{71} + \lambda_{l8}y_{81} - s_{l1}^{+} \ge y_{l1} - 0.00001 \times y_{l1},\\
&\lambda_{l1}y_{12} + \lambda_{l2}y_{22} + \lambda_{l3}y_{32} + \lambda_{l4}y_{42} + \lambda_{l5}y_{52} + \lambda_{l6}y_{62} + \lambda_{l7}y_{72} + \lambda_{l8}y_{82} - s_{l2}^{+} \ge y_{l2} - 0.00001 \times y_{l2},\\
&\lambda_{l1}y_{13} + \lambda_{l2}y_{23} + \lambda_{l3}y_{33} + \lambda_{l4}y_{43} + \lambda_{l5}y_{53} + \lambda_{l6}y_{63} + \lambda_{l7}y_{73} + \lambda_{l8}y_{83} - s_{l3}^{+} \ge y_{l3} - 0.00001 \times y_{l3},\\
&\lambda_{l1} + \lambda_{l2} + \lambda_{l3} + \lambda_{l4} + \lambda_{l5} + \lambda_{l6} + \lambda_{l7} + \lambda_{l8} = 1,\\
&s_{l1}^{+} \in \mathbb{Z},\ s_{l2}^{+} \in \mathbb{Z},\\
&s_{l1}^{-} \le 0.00001 \times x_{l1},\ s_{l2}^{-} \le 0.00001 \times x_{l2},\ s_{l3}^{-} \le 0.00001 \times x_{l3},\ s_{l4}^{-} \le 0.00001 \times x_{l4},\\
&\lambda_{l1} \ge 0,\ \lambda_{l2} \ge 0,\ \lambda_{l3} \ge 0,\ \lambda_{l4} \ge 0,\ \lambda_{l5} \ge 0,\ \lambda_{l6} \ge 0,\ \lambda_{l7} \ge 0,\ \lambda_{l8} \ge 0,\\
&s_{l1}^{-} \ge 0,\ s_{l2}^{-} \ge 0,\ s_{l3}^{-} \ge 0,\ s_{l4}^{-} \ge 0,\\
&s_{l1}^{+} \ge 0,\ s_{l2}^{+} \ge 0,\ s_{l3}^{+} \ge 0.
\end{aligned}$$

Note that, since the inputs are non-controllable data, KAM just considers negligible errors with 0.00001-DF and the input slacks can at most be optimized corresponding to the negligible errors. According to the following targets, user may only consider the same input values for each input target. The values of KAM targets with 0.0001-DF are given by:

$$\begin{aligned}
&x_{l1}^{*} = x_{l1} - s_{l1}^{-*} + 0.00001 \times x_{l1}, (\text{or } x_{l1}^{*} = x_{l1}),\\
&x_{l2}^{*} = x_{l2} - s_{l2}^{-*} + 0.00001 \times x_{l2}, (\text{or } x_{l2}^{*} = x_{l2}),\\
&x_{l3}^{*} = x_{l3} - s_{l3}^{-*} + 0.00001 \times x_{l3}, (\text{or } x_{l3}^{*} = x_{l3}),\\
&x_{l4}^{*} = x_{l4} - s_{l4}^{-*} + 0.00001 \times x_{l4}, (\text{or } x_{l4}^{*} = x_{l4}),\\
&\widehat{y_{l1}^{*}} = y_{l1} + s_{l1}^{+*}, (\text{or } y_{l1}^{*} = y_{l1} + s_{l1}^{+*} - \lfloor 0.00001 \times y_{l1} \rfloor),\\
&\widehat{y_{l2}^{*}} = y_{l2} + s_{l2}^{+*}, (\text{or } y_{l2}^{*} = y_{l2} + s_{l2}^{+*} - \lfloor 0.00001 \times y_{l2} \rfloor),\\
&y_{l3}^{*} = y_{l3} + s_{l3}^{+*} - 0.0001 \times y_{l3}.
\end{aligned}$$

From the above illustration, KAM provides a flexible methodology to assess the performance evaluation of DMUs easily according to the available information.

Note that, linear KAM gives the optimum slacks, but may not give the minimum scores (Khezrimotlagh et al., 2013c). In order to measure optimum scores, it is suggested to use non-linear KAM by the following objective subject to the constraints of Equation (1) which can be transformed to linear form easily.

$$\min \frac{1 + \varepsilon - \frac{1}{4}\left(\frac{s_{l1}^{-}}{x_{l1}} + \frac{s_{l2}^{-}}{x_{l2}} + \frac{s_{l3}^{-}}{x_{l3}} + \frac{s_{l4}^{-}}{x_{l4}}\right)}{1 - \varepsilon + \frac{1}{3}\left(\frac{s_{l1}^{+}}{y_{l1}} + \frac{s_{l2}^{+}}{y_{l2}} + \frac{s_{l3}^{+}}{y_{l3}}\right)}.$$

In this case, the score of non-linear KAM is always less than or equal to the score of linear KAM. From this illustration, it does not deduce that the score of linear KAM is always decreasing as epsilon increases.

But, the score of the above non-linear KAM is always decreasing by increasing the values of epsilons. When epsilon is zero, the above non-linear KAM is Slack-Based Measure (SBM) (Tone, 2001).

RESULTS

Table 1 shows the used data in this practice as well as the results of BCC and SBM-Non-Oriented-VRS models.

Table 1: International Iranian Airports in 2009.

DMUs	Area	Apron	Terminal	Runway	Passengers	Cargo	Flights	Rank by BCC	Rank by SBM-VRS
OIFM	1,041	112,464	21,050	395,730	1,744,524	4,919	39,871	1	1
OIIE	1,200	304,182	45600	353,610	4,030,859	74,184	30,707	1	1
OIKB	481	47,210	9,300	268,995	971,313	3,826	19,010	1	1
OITT	800	41,003	11,800	269,955	1,039,967	1,587	15,608	1	1
OIZH	1,002	30,000	8,000	192,330	427,974	1,574	4,887	1	1
OISS	478	63,000	23,000	389,115	2,165,572	5,414	41,088	1	1
OIMM	503	213,729	38,778	348,120	4,783,120	19,050	46,875	1	1
OIII	1,346	503,274	76,370	421,305	11,709,741	39,556	129,153	1	1

Table 2 shows the ranks and efficiency scores of 0.00001-KAM for three mentioned scenarios.

Table 2: Rank and efficiency scores of airports by linear KAM VRS with 0.00001-DF.

DMU	Data are real.		The first two outputs are integer.		Inputs are non-controllable and the first two outputs are integer.	
	Rank	Scores	Rank	Scores		
OIFM	8	0.999722	8	0.999782	8	0.999853
OIIE	5	0.999981	6	0.999974	6	0.999974
OIKB	4	0.999992	4	0.999995	4	0.999995
OITT	7	0.999901	5	0.999985	5	0.999990
OIZH	6	0.999968	7	0.999972	7	0.999974
OISS	3	0.999992	1	0.999997	1	0.999997
OIMM	2	0.999998	1	0.999997	1	0.999997
OIII	1	0.999999	3	0.999995	1	0.999997

Note that, as the number of constraints increase, the scores should be also increased. However, the score of OIII, for instance, is not increasing in Table 2. This is due to our assumptions for targets in each scenario. Indeed, when the targets are restricted to the set of integer numbers set, linear KAM suggests the points in the efficient tape with 0.00001-DF which may not be on the Farrell frontier nor in the first PPS, but it is very close to them by 0.00001-DF. Moreover, the targets are considered the same for each DMU and therefore, the scores in each column are appropriate to distinguish between airports. If user would concern about decreasing the scores, and would want to find minimum scores, it is suggested to use non-liner KAM.

For distinguish between OISS, OIMM and OIII in the third scenario it is enough to consider one more decimal digit for each score which are 0.9999972, 0.9999974, 0.9999968, respectively.

CONCLUSIONS

In this paper, a numerical example to depict the advantages of applying KAM is proposed. KAM is able to distinguish between technically efficient DMUs even if the number of DMUs is less than the number of variables. Moreover, KAM is flexible to handle different data while the constraints are increased.

REFERENCES

[1] Banker, R.D., Charnes A., Cooper, W.W. (1984) Some Models for Estimating Technical and scale Inefficiencies in Data Envelopment Analysis. *Management Science* 30(9): 1078-1092.

[2] Charnes A., Cooper W.W., Rhodes E. (1978) Measuring the efficiency of decision-making units, *European Journal of Operational Research* 2(6): 429-444.

[3] Charnes, A., Cooper W.W., Golany, B., Seiford, L.M., Stutz, J. (1985) Foundations of data envelopment analysis and Pareto-Koopmans empirical production functions. *Journal of Econometrics* 30(1): 91-107.

[4] Khezrimotlagh, D., Salleh, S., Mohsenpour, Z. (2012a) A New Method in Data Envelopment Analysis to Find Efficient Decision Making Units and Rank Both Technical Efficient and Inefficient DMUs Together. *Applied Mathematical Sciences* 6(93): 4609-4615.

[5] Khezrimotlagh, D., Mohsenpour, Z., Salleh, S. (2012b) Cost-Efficiency by Arash Method in DEA. *Applied Mathematical Sciences* 6(104): 5179-5184.

[6] Khezrimotlagh, D., Salleh, S., Mohsenpour, Z. (2012c) Arash Method and Uncontrollable Data in DEA. *Applied Mathematical Sciences* 6(116): 5763-5768.

[7] Khezrimotlagh, D., Mohsenpour, Z., Salleh, S. (2012d) Comparing Arash Model with SBM in DEA. *Applied Mathematical Sciences* 6(104): 5185-5190.

[8] Khezrimotlagh, D., Salleh, S., Mohsenpour, Z. (2012e) Airport Efficiency with Arash Method in DEA. *Journal of Basic and Applied Scientific Research* 2(12): 12502-12507.

[9] Khezrimotlagh, D., Salleh, S., Mohsenpour, Z. (2012f), Benchmarking Inefficient Decision Making Units in DEA, *Journal of Basic and Applied Scientific Research*, 2(12): 12056-12065.

[10] Khezrimotlagh, D., Salleh S., Mohsenpour, Z. (2013a) A New Method for Evaluating Decision Making Units in DEA, *Journal of the Operational Research Society*, 65(1): 694–707.

[11] Khezrimotlagh, D., Salleh, S., Mohsenpour, Z. (2013b) A New Robust Mixed Integer Valued Model in DEA, *Applied Mathematical Modelling*, 37(24): 9885-9897.

[12] Khezrimotlagh, D., Salleh, S., Mohsenpour, Z. (2013c) Nonlinear Arash Model in DEA. *Research Journal of Applied Sciences. Engineering and Technology* 5(17): 4268-4273.

[13] Khezrimotlagh, D., Mohsenpour, P., Salleh, S., Mohsenpour, Z. (2013d). Decision Making Units with Integer Values in Data Envelopment Analysis. *In: Banker B., A. Emrouznejad, H. Bal, I. Alp, M. Ali Gengiz (2013), Data Envelopment Analysis and Performance Measurement: Proceedings of the 11th International Conference of DEA, June 2013, Samsun, Turkey, ISBN: 9781854494771*.

[14] Khezrimotlagh, D., Mohsenpour, P., Salleh, S., Mohsenpour, Z. (2013e). A Review on Arash Method in Data Envelopment Analysis. *In: Banker B., A. Emrouznejad, H. Bal, I. Alp, M. Ali Gengiz (2013), Data Envelopment Analysis and Performance Measurement: Proceedings of the 11th International Conference of DEA, June 2013, Samsun, Turkey, ISBN: 9781854494771*.

[15] Khezrimotlagh, D., (2014). How to Select an Epsilon in Kourosh and Arash Model in DEA. *The 12th International Conference of DEA, April 2014, Kuala Lumpur, Malaysia.*

[16] Lozano, S., Gutierrez, E. (2011) Slacks-based measure of efficiency of airports with airplanes delays as undesirable outputs. *Computers and Operations Research* 38(1): 131-139.

[17] Tone, K. (2001) A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research* 130(3): 498-509.