Assessing Preservice Teachers' Knowledge of the Relationship between Perimeter and Area

Wun Thiam Yew[1], Sharifah Norul Akmar Syed Zamri[2], Lim Hooi Lian[1] and Chew Cheng Meng[1]

[1]School of Educational Studies, Universiti Sains Malaysia
[2]Faculty of Education, University of Malaya

Abstract. This paper attempts to assess preservice teachers' knowledge of the relationship between perimeter and area. In this study, the researchers employed survey research design to assess preservice teachers' knowledge of the relationship between perimeter and area. A questionnaire was employed to collect the data. Convenient sampling technique was employed to select the participants of the study. Respondents of the study consisted of 34 preservice teachers (majored or minored in mathematics) who are attending Bachelor of Science with Education program at a public university in Peninsula Malaysia. This paper presents the analysis of the responses of the preservice teachers related to a particular mathematical task, namely Same perimeter, same area? The finding suggests that 73.53% of the preservice teachers in this study provided the correct response that the student's method of calculating the area of the leaf was not correct. They knew that there is no direct relationship between perimeter and area. The preservice teachers knew that two shapes with the same perimeter can have different areas. Thus, they knew that the student's method of calculating the area of the leaf was not correct. Implications of the findings were also discussed.

Keywords: *preservice teachers, relationship between perimeter and area, survey research design, convenient sampling technique*

1 Introduction

Ferrer, Hunter, Irwin, Sheldon, Thompson, and Vistro-Yu (2001) observed that students in many parts of the world encountered difficulty in understanding the concepts of perimeter and area. It is even harder to fully understand the nonconstant relationship between perimeter and area (Ferrer et al., 2001). Bennett and Nelson (2001) pointed out that:

> "Intuitively, it may seem that the area of a region should depend on its perimeter. For example, if a person uses more fences to close in a piece of land than another person, it is tempting to assume the first person has enclosed the greater amount of land. However, this is not necessarily true." (p. 658)

Thus, two shapes with the same perimeter could have different areas. Similarly, two shapes with the same area could have different perimeters. For a given perimeter, the dimensions of a shape affect its area. For instance, for a given perimeter of 20 cm, the area of a rectangle could be 9 cm × 1 cm = 9 cm^2, 8 cm × 2 cm = 16 cm^2, and so on.

There is no direct relationship between perimeter and area (Ball, 1988; Haylock, 2001). Nevertheless, previous studies revealed that many students and prospective teachers had a misconception that there is direct relationship between perimeter and area (Arnold, Turner, & Cooney, 1996; Ball, 1988; Chappell, & Thompson, 1999; Tierney, Boyd, & Davis, 1990; Woodward, 1982; Woodward & Byrd, 1983; Wun & Sharifah Norul Akmar, 2010). They thought that two shapes with the same perimeter have the same area.

Woodward (1982) found that an excellent seventh grade student, Heidi, thought that the garden with the same perimeter have the same area. Woodward and Byrd (1983) revealed that 59% of the 129 eight grade students at a junior high school in Tennessee thought that the garden with the same perimeter have the same area. They also revealed that 63% of the 129 eight grade students at another junior high school in Tennessee thought that the garden with the same perimeter have the same area. Woodward and Byrd (1983) found that prospective elementary teachers took the test with similar results.

Arnold et al. (1996) revealed that most of the middle school and university students in their study thought that when the perimeter of a shape is held constant, its area remains constant. Likewise, Chappell and Thompson (1999) found that only one out of 29 (i.e., 3%) grade six students in their study were able to justify that two shapes with the same area could have different perimeters. None of the 19 grade seven students in their study were able to justify that two shapes with the same area could have different perimeters while three out of 16 (i.e., 19%) grade eight students in their study were able to justify that two shapes with the same area could have different perimeters.

Wun & Sharifah Norul Akmar (2010) found that only one of the eight preservice teachers in their study, namely Suhana, provided the correct response that the student's method of calculating the area of the leaf was not correct. She knew that there is no direct relationship between perimeter and area. Suhana knew that two shapes with the same perimeter can have different areas. Thus, she knew that the student's method of calculating the area of the leaf was not correct. Five of the preservice teachers, namely Mazlan, Roslina, Patrick, Tan, and Usha, thought that the student's method of calculating the area of the leaf was correct. They did not know that there is no direct relationship between perimeter and area. Mazlan, Roslina, Patrick, Tan, and Usha did not know that two shapes with the same perimeter can have different areas. Thus, they thought that the student's method of calculating the area of the leaf was correct. The remaining two preservice teachers, namely Beng and Liana, were not sure whether the student's method of calculating the area of the leaf was correct or not. They did not know that there is no direct relationship between perimeter and area. Beng and Liana did not know that two shapes with the same perimeter can have different areas. Thus, they were not sure whether the student's method of calculating the area of the leaf was correct or not.

This paper attempts to assess preservice teachers' knowledge of the relationship between perimeter and area.

2 Methodology

In this study, the researchers employed survey research design to assess preservice teachers' knowledge of the relationship between perimeter and area. Convenient sampling technique was employed to select the participants of the study. Respondents of the study consisted of 34 preservice teachers (majored or minored in mathematics) who are attending Bachelor of Science with Education program at a public university in Peninsula Malaysia.

This paper reports only the responses of the participants on Task 5.1 (see Appendix A). The task was adapted from previous study (Wilson & Chavarria, 1993, pp. 139-140). In this task, a Form One student claimed that he found a way to calculate the area of a leaf. The student placed a piece of thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle. Subjects were asked how they would respond to this student. The objective of this task was to determine the subjects' knowledge of the relationships between perimeter and area of an irregular figure. This task was used to ascertain whether the subjects knew that there is no direct relationship between perimeter and area. Two shapes with the same perimeter can have different areas.

A questionnaire was employed to collect the data. The questionnaire was administered to all the preservice teachers (majored or minored in mathematics) who are attending Bachelor of Science with Education program at a public university in Peninsula Malaysia.

3 Findings of the Study

The finding suggests that 25 (i.e., 73.53%) of the preservice teachers in this study provided the correct response that the student's method of calculating the area of the leaf was not correct. They knew that there is no direct relationship between perimeter and area. The preservice teachers knew that two shapes with the same perimeter can have different areas. Thus, they knew that the student's method of calculating the area of the leaf was not correct.

12 of these 25 preservice teachers were able to generate a counterexample to show that the student's method of calculating the area of the leaf was not correct. Figures 1 through 5 show the selected counterexamples generated by these preservice teachers:

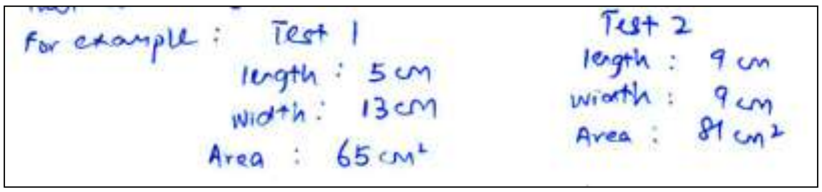


Figure 1. Counterexample generated by R1.

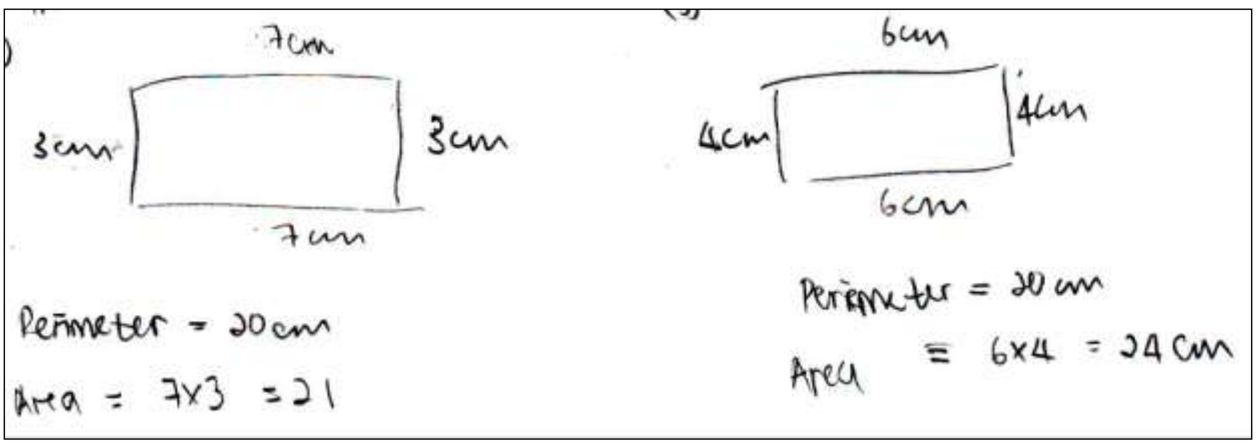


Figure 2. Counterexample generated by R6.

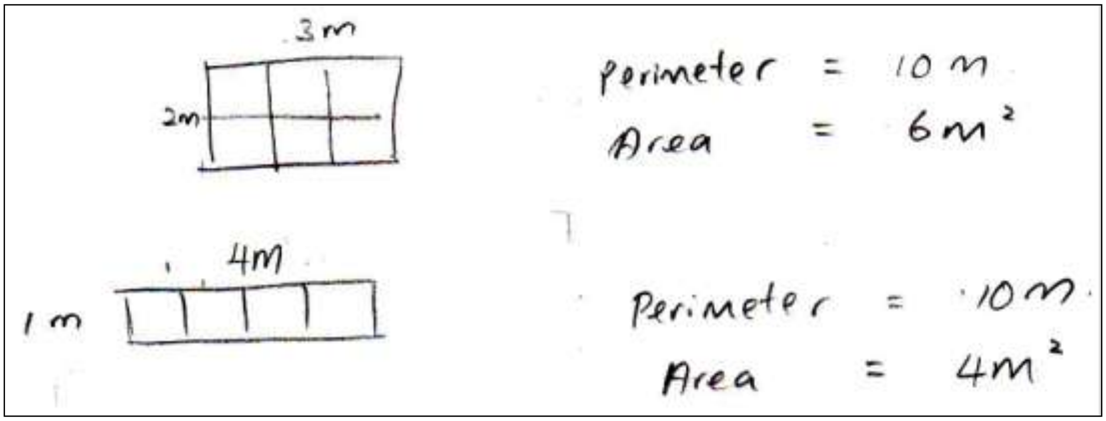


Figure 3. Counterexample generated by R7.

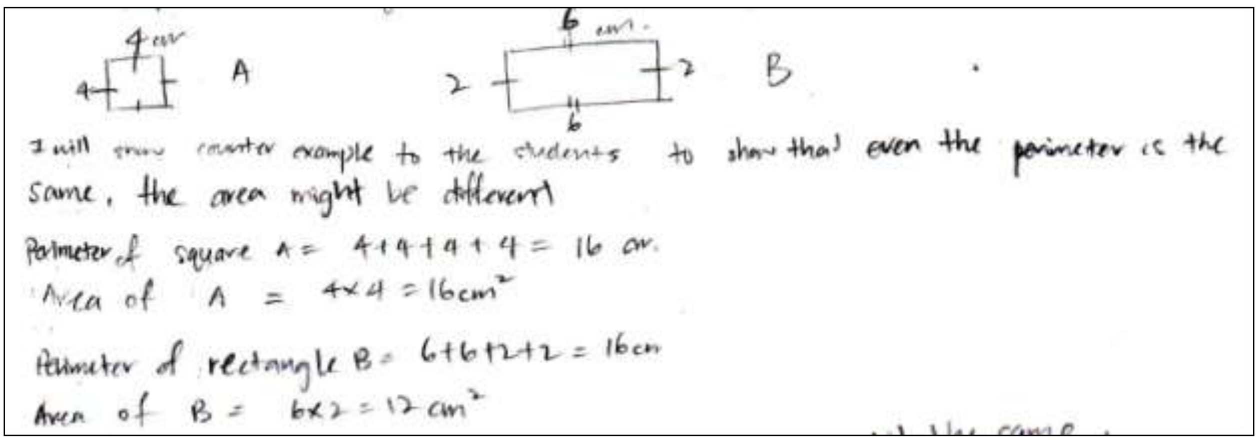


Figure 4. Counterexample generated by R8.

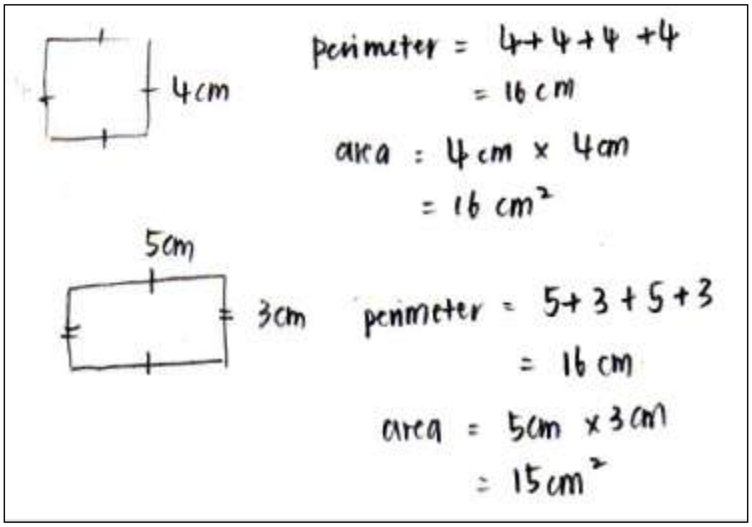


Figure 5. Counterexample generated by R34.

The remaining 9 (i.e., 26.47%) of the preservice teachers in this study thought that the student's method of calculating the area of the leaf was correct. They did not know that there is no direct relationship between perimeter and area. The preservice teachers did not know that two shapes with the same perimeter can have different areas. Thus, they thought that the student's method of calculating the area of the leaf was correct. Below are the selected written responses from these preservice teachers:

> "This method is creative. … area of rectangle = area of the leaf = length × width of the rectangle." (R3)
>
> "Yes. Awesome! So clever! …" (R13)
>
> "The method used is correct. Area of the rectangle is equal to the area bounded by the thread which surround the leaf." (R14)
>
> "Good! The method you used is correct. The student is able to construct their own ways of learning on finding the area of irregular shape." (R16)
>
> "Yes, you have come with a great idea. We can use that way to calculate the area …" (R20)
>
> "Yes, you are correct and have the creative thinking …" (R21)
>
> "Area of the rectangle = ab. The area of the leaf = summation of area of rectangle." (R23)
>
> "…I appreciate and complement him/her for being creative and able to think the alternative solution." (R24)
>
> "Yes, you can do like that to find the area of the leaf. …" (R25).

4 Discussion and Conclusions

The finding suggests that 73.53% of the preservice teachers in this study knew that there is no direct relationship between perimeter and area. The preservice teachers knew that two shapes with the same perimeter can have different areas. Thus, they knew that the student's method of calculating the area of the leaf was not correct. This finding is in contrast with the findings of previous studies (Arnold, Turner, & Cooney, 1996; Chappell & Thompson, 1999; Woodward, 1982; Woodward & Byrd, 1983; Wun & Sharifah Norul Akmar, 2010).

The finding suggests that 26.47% of the preservice teachers in this study had a misconception that there is direct relationship between perimeter and area. They thought that two shapes with the same perimeter have the same area. However, this study only involved 34 preservice teachers. They enrolled in the 4-year Bachelor of Science with Education (B.Sc.Ed.) program in a public university in Peninsula Malaysia. Convenient sampling technique was employed to select the participants of the study. Thus, the findings of this study could not be generalized to other preservice teachers enrolled in the 4-year Bachelor of Science with Education (B.Sc.Ed.) program in this public university, in other programs (e.g., Bachelor of Education (B. Ed.), Diploma in Education (Dip.Ed.)), or attending other universities and teacher training institutes.

The implication of this finding is that mathematics teacher educators need to organize teaching and learning activities that provide opportunity for the preservice mathematics teachers to use unit square chips or tiles to examine the possible pattern of relationship between perimeter and area, formulate and test generalizations pertaining to the relationship between perimeter and area, such as areas of shapes having the same perimeter and vice versa. Through such activities, preservice mathematics teachers would understand that there is no direct relationship between perimeter and area. They would know that two shapes with the same perimeter could have different areas.

This is in line with the recommendations in the Form One Mathematics Curriculum Specifications (Ministry of Education Malaysia, 2003) which suggests that the mathematics educators need to provide opportunity for their students to use unit square chips or tiles to investigate, explore, and make generalizations about the: (a) "perimeters of rectangles having the same area; and (b) areas of rectangles having the same perimeter" (p. 42). As mathematics teacher educators, we need to search for appropriate situations that would enable the preservice teachers to discover and construct the different aspects of the topics that they will teach and ultimately, facilitate them to do the same for their future students.

References

Arnold, B., Turner, P., & Cooney, T. J. (1996). Analyzing teaching and learning: The art of listening. *Mathematics Teacher, 89*(4), 326-329.

Ball, D. L. (1988). *Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education.* Unpublished doctoral dissertation, Michigan State University, East Lansing.

Bennett, A. B. Jr., & Nelson, L. T. (2001). *Mathematics for elementary teachers: A conceptual approach* (5th ed.). New York: McGraw-Hill.

Chappell, M., & Thompson, D. R. (1999). Perimeter or area? Which measure is it? *Mathematics Teaching in the Middle School, 5*(1), 20-23.

Ferrer, B., Hunter, B., Irwin, K., Sheldon, M., Thompson, C., & Vistro-Yu, C. (2001). By the linear unit or square unit? *Mathematics Teaching in the Middle School, 7*(3), 132-139.

Haylock, D. (2001). *Mathematics explained for primary teachers* (2nd ed.). London: Paul Chapman Publishing.

Ministry of Education Malaysia (2003). *Integrated Curriculum for Secondary School: Curriculum Specification Mathematics Form 1.* Kuala Lumpur: Curriculum Development Centre.

Tierney, C., Boyd, C., & Davis, G. (1990). Prospective primary teachers' conceptions of area. In G. Booker, P. Cobb, & T. de Mendicuti (Eds.), *Proceedings of the Fourteenth Annual Conference of the International Group for* *the Psychology Mathematics Education with North American Chapter* (Vol. 2, pp. 307-315), Mexico. (ERIC Document Reproduction Service No. ED411138).

Wilson, L. D., & Chavarria, S. (1993). Superitem tests as a classroom assessment tool. In N. L. Webb & A. F. Coxford (Eds.), *Assessment in the mathematics classroom: 1993 yearbook.* Reston, VA: National Council of Teachers of Mathematics.

Woodward, E. (1982). Heidi's misconception about area and perimeter. *School Science and Mathematics, 82*(4), 332-334.

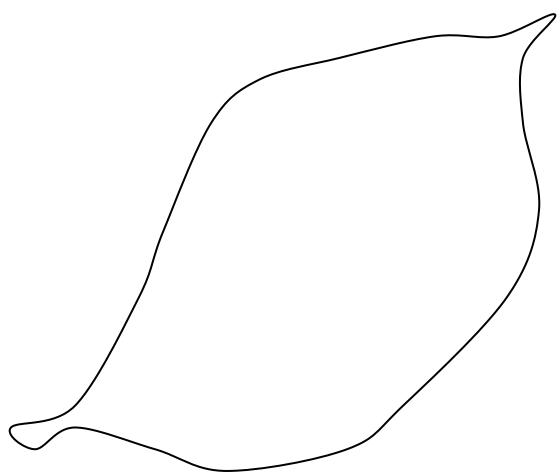
Woodward, E., & Byrd, F. (1983). Area: Included topic, neglected concept. *School Science and Mathematics, 83*(4), 343-347.

Wun, T. Y., & Sharifah Norul Akmar, S. Z. (2010). Preservice teachers' knowledge of the relationship between perimeter and area. *Malaysian Education Deans' Council Journal, 5*, 136-145.

Appendix A

Task 5.1: Same perimeter, same area? (*Adapted from Wilson & Chavarria, 1993, pp. 139-140*)

This is a picture of a leaf. A Form One student said that he had found a way to calculate the area of the leaf. The student placed a piece of thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle.



How would you respond to this student?