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ASSESSING PRESERVICE SECONDARY SCHOOL MATHEMATICS TEACHERS' KNOWLEDGE OF AREA

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Abstract

The purpose of this paper is to assess preservice secondary school mathematics teachers' knowledge of area. Clinical interview technique was employed to collect the data. Subjects of this study consisted of eight preservice secondary school mathematics teachers enrolled in a Mathematics Teaching Methods course at a public university in Peninsula Malaysia. They were selected based on their majors (mathematics, biology, chemistry, physics) and minors (mathematics, biology, chemistry, physics). This paper presents the analysis of the responses of the subjects related to a particular task, namely notion of area. The finding suggests that five of the preservice teachers in this study had the correct notion of area that 2-dimensional shapes (closed plane shapes) and 3-dimensional shapes have an area. Different categories of incorrect notion of area were identified. Preservice teachers' linguistic knowledge and ethical knowledge of area were discussed. The implication of this finding was also discussed.

Keywords: preservice secondary school mathematics teachers; knowledge of area; case study; clinical interview

Introduction

Teachers must have in-depth knowledge of mathematics they are going to teach. Therefore, it is important that teachers need to have a comprehensive knowledge of mathematics to enable them to organize teaching so that students can learn mathematics meaningfully. Fennema and Franke (1992) advocated that "No one questions the idea that what a teacher know is one of the most important influences on what is done in classroom and ultimately on what students learn" (p. 147). Furthermore, "Teachers who do not themselves know a subject well are not likely to help students learn this content." (Ball, Thames, & Phelps, 2008, p. 404). This applies also to mathematics teacher.

Notion of Area

Numerous definitions of area were provided by the researchers or mathematics educators. Table 1 shows some of these definitions. Martin and Strutchens (2000) noted that "The concept of area is often difficult for students to understand, perhaps due to their initial experiences in which it is tied to a formula (such as area = length × width) rather than more conceptual activities such as counting the number of square units it would take to cover a surface" (p. 223). Cavanagh (2008) found that 53% of the 43 Year 7 students from two government high schools in Sydney in his study defined area as 'space inside the shape' while 19% referred it as 'length by width'. However, Tierney, Boyd, and Davis (1990) revealed that many prospective primary school teachers from a teachers college in their study thought that area is 'length by width'. When the prospective teachers were asked what they would teach a ten year old child about area, "80% of them drew a rectangle and wrote "$l \times w$" or "l by w" near it. Some of these prospective teachers placed arrows aroud a rectangle in a way which denoted perimeter rather than area" (pp. 307-308). The remaining 20% of prospective teachers defined area as the space inside a figure. Furthermore, Casa, Spinelli, and Gavin (2006) noticed that many adults thought that area is 'length by width'. "They understand area as a formula rather than as a concept - the amount of space covered by the inside boundaries of a two-dimensional figure" (Casa et al., 2006, p. 168).

Table 1
Some of the definitions of area

Researchers or mathematics educators	Definition of area
Ball, 1988, p. 170.	The area is the number of unit squares it takes to cover the figure or region.
Bennett & Nelson, 2001, p. 653.	The number of units it takes to cover a surface (or region) is called its area.
Billstein, Liberskind, & Lott, 2006, p. 750.	Area of a region is the number of nonoverlapping square units that covers the region.
Cathcart, Pothier, Vance, & Bezuk, 2006, p. 330.	Area is the amount of surface enclosed by a curve in the plane.
Haylock, 2001, p. 268.	Area is a measure of the amount of two-dimensional space inside a boundary.
Long & DeTemple, 2003, p. 771.	The number of units required to cover a region in

	the plane is the area of the region.
Rickard, 1996, p. 306.	Area is represented as the number of square units needed to cover a shape.
Suggate, Davis, & Goulding, 1999, p. 134.	Area-amount of surface.

Baturo and Nason (1996) suggested that area can be viewed from two different perspectives, namely static and dynamic perspectives. From the static perspective, area can be viewed as the amount of surface enclosed within a boundary. If a preservice teacher selected one or more open shapes and explained that the shape(s) had an area of zero, then it indicated that the preservice teacher is having a dynamic perspective of area. Baturo and Nason (1996) found that none of the 13 preservice primary school teachers in their study selected open shapes (including the lines) as having an area. It can be inferred that they did not have a dynamic perspective of the notion of area. Furthermore, all of them indicated that these shapes (i.e., open shapes) needed to be closed showing that they had a static perspective of the notion of area. Baturo and Nason (1996) also found that three of the preservice teachers in their study appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured).

Wun and Lim (2011) revealed that 36% of the preservice special education teachers in their study had the correct notion of area that 2-dimensional shapes (closed plane shapes) and 3-dimensional shapes have an area. The study of Wun, Lim, and Chew (2012) showed that 78.26% of the preservice teachers in their study have successfully selected all the shapes that have an area. They had the correct notion of area that 2-dimensional shapes (closed plane shapes) and 3-dimensional shapes have an area. Review of research literature had also shown that some students and preservice teachers encountered difficulty in differentiating between the attributes of perimeter, area, and volume (Baturo & Nason, 1996; Beaumont, Curtis, & Smart, 1986; Ramakrishnan, 1998; Reinke, 1997; Wun & Lim, 2011; Wun, Lim, & Chew, 2012).

Purpose of the Study

The purpose of this study was to assess preservice secondary school mathematics teachers (PSSMTs)' knowledge of a specific mathematical topic, namely perimeter and area, in particular, on the notion of area. Specifically, this study aimed to assess PSSMTs' conceptual knowledge, linguistic knowledge, and ethical knowledge of area.

Conceptual Framework of the Study

Nik Azis (1996) suggested that there are five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge. In the present study, the researchers have adapted Nik Azis's (1996) categorization of knowledge to assess PSSMTs' knowledge of area.

Methodology

The discussion about the methodology of this study consisted of five sections: research design, selection of subjects, instrumentation, data collection, and data analysis.

Research Design

In this study, the researchers employed case study research design to assess, in-depth, PSSMTs' knowledge of area. "A case study design is used to gain an in-depth understanding of the situation and meaning for those involved" (Merriam, 1998, p. 19). Several researchers (e.g., Aida Suraya, 1996; Chew, 2007; Lim, 2007; Rokiah, 1998; Seow, 1989; Sharifah Norul Akmar, 1997; Sutriyono, 1997; Wun, 2010) employed case study research design to study Malaysian students, preservice teachers, and lecturers.

Selection of Subjects

The researchers employed purposeful sampling to select the subjects (sample) for this study. Eight subjects were selected for the purpose of this study. They were PSSMTs from a public university in Peninsula Malaysia enrolled in a 4-year Bachelor of Science with Education (B.Sc.Ed.) program, majored or minored in mathematics. These subjects enrolled for a one-semester mathematics teaching methods course during the data collection of this study. The mathematics teaching methods course was offered to B.Sc.Ed. program students who intended to major or minor in mathematics. The researchers had selected four B.Sc.Ed. program students who majored in mathematics, and four B.Sc.Ed. program students who minored in mathematics for the purpose of this study. They do not have any teaching experience prior to this study. Each PSSMTs was given a pseudonym, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, in order to protect the anonymity of all interviewees. The brief background information about the subjects is shown in Table 2.

Table 2
Subjects' Ethnicity, Gender, Age, Major, Minor, and CGPA

Subject	Ethnicity	Gender	Age	Major	Minor	CGPA
Usha	Indian	Female	(21, 9)	Mathematics	Biology	2.92
Mazlan	Malay	Male	(21, 8)	Mathematics	Chemistry	2.70
Patrick	Bidayuh	Male	(21, 7)	Mathematics	Chemistry	3.04
Beng	Chinese	Female	(22, 9)	Mathematics	Physics	3.82
Roslina	Malay	Female	(21, 8)	Biology	Mathematics	3.15
Liana	Malay	Female	(21, 5)	Chemistry	Mathematics	2.77
Tan	Chinese	Male	(22, 7)	Chemistry	Mathematics	3.69

Suhana	Malay	Female	(20, 10)	Physics	Mathematics	2.52

Instrumentation

This paper reports only the responses of the participants on Task 1.2 (see Appendix A). This task was adapted from previous study (Baturo & Nason, 1996, p. 245). In Task 1.2, the respondents were asked to select the shapes (12 shapes) that have an area. The objective of this task was to determine the participants' conceptual knowledge about the notion of area. Six 2-dimensional shapes (labelled as A, C, D, H, I, K in Appendix A) were used to ascertain whether the respondents understood area from a static perspective. Based on this perspective, "area can be considered as the amount of surface enclosed within a boundary" (Baturo & Nason, 1996, p. 245). Two open shapes (labelled B, G in Appendix A) were included to investigate further the participants' understanding of the notion of area from a static perspective.

Two 1-dimensional shapes (E, L) were included to ascertain whether the respondents understood area from a dynamic perspective. If the participants selected one or both of these shapes and explained that the shape(s) had an area of zero, then this response indicated that the respondents are having a dynamic perspective of area (Baturo & Nason, 1996). Finally, two 3-dimensional shapes (F, J) were included because review of research literature had shown that some students and preservice teachers encountered difficulty in differentiating between the attributes of perimeter, area and volume (Baturo & Nason, 1996; Beaumont, Curtis, & Smart, 1986; Ramakrishnan, 1998; Reinke, 1997; Wun & Lim, 2011 Wun, Lim, & Chew, 2012).

Task 1.2 was also used to determine the participants' linguistic knowledge of area based on the language of mathematics (such as mathematical terms and symbols) that the subjects used to justify the selection of shapes that have an area. There are some good behaviors that the respondents needed to follow when dealing with area. Knowledge and justification of knowledge is an important aspect in any discipline. Thus, this task was also used to determine the participants' ethical knowledge of area by ascertaining whether the respondents justify the selection of shapes that have an area.

Data Collection

Data for this study was collected using clinical interview techniques. The interview was conducted in the Mathematics Teaching Room at a public university in Peninsula Malaysia. The physical setting for each interview consisted of a table with two chairs, a tape recorder and a digital video camera. Each interview was recorded through the tape recorder and digital video camera positioned in front of the table. The camera was focused on the subject, the working area, and the researcher. Blank papers, grid papers, pencil, ruler, thread, compasses, and calculator were accessible to the subject throughout the interview. Materials collected for analysis consisted of audiotapes and videotapes of clinical interview, subject's notes and drawings, and researcher's notes during the interview.

Data Analysis
The data analysis process encompassed four levels. At level one, the audio and video recording of the clinical interview were verbatim transcribed into written form. The transcription included the interaction between the researcher and the subject during the interviews as well as the subject's nonverbal behaviors. At level two, raw data in the forms of transcription were coded, categorized, and analyzed according to specific themes to produce protocol related to the description of the subjects' knowledge of area.

At level three, case study for each subject was constructed based on information from the written protocol. At this level, analysis was carried out to describe each subject's behaviors in solving every tasks or problems. At level four, cross-case analysis was conducted. The analysis aimed to identify pattern of responses of knowledge of area held by the subjects. Based on this pattern of responses, PSSMTs' knowledge of area were summarized.

Findings of the Study

In this section, findings of PSSMTs' knowledge of area was presented in terms of its components as stipulated in the previous section.

Conceptual Knowledge
In this study, five of the eight PSSMTs in this study, namely Liana, Mazlan, Suhana, Tan, and Usha, have successfully selected all the shapes (A, C, D, F, H, I, J, K)
that have an area. They had the correct notion of area that 2-dimensional shapes (closed plane shapes) (A, C, D, H, I, K) and 3-dimensional shapes (F, J) have an area. It revealed that they had a static perspective of the notion of area. Based on this perspective, area can be viewed as the amount of surface enclosed within a boundary. Table 3 depicts each PSSMT's selection of shapes that have an area and their notion of area.

Table 3
PSSMTs' Selection of Shapes That Have an Area and Their Notion of Area

Selection of shapes that have a area	Notion of area	PSSMTs
A, C, D, F, H, I, J, K	2-dimensional shapes and 3-dimensional shapes	Liana, Mazlan, Suhana, Tan, Usha
A, C, D, H, I, K	Limited to 2-dimensional shapes	Beng
A, C, F, H, J	Limited to regular 2-dimensional shapes (such as triangle, circle, and trapezium) and 3-	Patrick, Roslina

	dimensional shapes (such as cuboid and cylinder)	

One PSSMT, namely Beng, had the incorrect notion of area that only 2-dimensional shapes (A, C, D, H, I, K) have an area. It revealed that she had a static perspective of the notion of area. Based on this perspective, area can be viewed as the amount of surface enclosed within a boundary. The remaining two PSSMTs, namely Patrick and Roslina, had the incorrect notion of area that only regular 2-dimensional shapes (such as triangle, circle, and trapezium) (A, C, H) and 3-dimensional shapes (such as cuboid and cylinder) (F, J) have an area.

All the PSSMTs in this study did not select the two open shapes (B, G) as well as the two 1-dimensional shapes (E, L) that do not have an area. In other words, they did not select an open shape (including the lines) as having an area. It can be inferred that all the PSSMTs did not have a dynamic perspective of area or this knowledge was not accessible to them during the clinical interview.

Linguistic Knowledge

When asked to justify their selection of shapes that have an area, the finding suggests that all the PSSMTs who had selected shapes A, C, D, F, H, I, J, and K that have an area used appropriate mathematical terms to justify their selection of shapes that have an area as follow, except Liana: (a) closed, (b) enclosed, (c) (can be) calculated, or (d) 3D. Liana used inappropriate word 'joining' to justify her selection of shapes A, C, D, F, H, I, J, and K that have an area. Five PSSMTs, namely Beng, Patrick, Roslina, Tan, and Usha, appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Table 4 demonstrates PSSMTs' selection of shapes that have an area and the appropriateness of their justification.

Table 4

PSSMTs' Selection of Shapes That Have an Area and the Appropriateness of Their Justification

Selection of shapes that have an area	Justification		PSSMTs
	Appropriate	Inappropriate	
A	Closed		Usha
	Closed shape		Mazlan, Suhana
	Closed length object		Tan
	Enclosed		Beng
	Its area can be calculated		Patrick, Roslina
		The lines are joining	Liana
C, H	Closed		Usha

	Closed shape		Mazlan, Suhana
	Its area can be calculated		Beng, Patrick, Roslina, Tan
		The lines are joining	Liana
D	Closed		Usha
	Closed shape		Mazlan, Suhana
	Its area can be calculated		Beng, Tan
		The lines are joining	Liana
F, J	3D object		Mazlan
	3D has surface area		Suhana
	Its surface area can be calculated		Patrick, Roslina, Tan, Usha
		The lines are joining	Liana
I	Closed shape		Mazlan, Suhana
	Enclosed		Beng
	Its area can be calculated		Tan, Usha
		The lines are joining	Liana
K	Closed		Usha
	Closed shape		Mazlan, Suhana
	Enclosed		Beng
	Its area can be calculated		Tan
		The lines are joining	Liana

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. The finding suggests that all the PSSMTs had taken the effort to justify the selection of shapes that have an area. All the PSSMTs who had selected shapes A, C, D, F, H, I, J, and K that have an area provided appropriate justification for their selection, except Liana. She had provided inappropriate justification for selecting these shapes that have an area, as shown in Table 4.

Conclusion

In conclusion, five of the eight PSSMTs in this study, namely Liana, Mazlan, Suhana, Tan, and Usha, had the correct notion of area that 2-dimensional shapes (closed plane shapes) (A, C, D, H, I, K) and 3-dimensional shapes (F, J) have an area. This finding is in contrast with the finding of Wun and Lim's (2011) study which revealed that 36% of the preservice special education teachers in their study had the correct notion of area that 2-dimensional shapes (closed plane shapes) and 3-dimensional shapes have an area. This finding is also in contrast with the finding of Wun, Lim, and Chew's (2012) study which found that 78.26% of the preservice teachers in their study had the correct notion of area that 2-dimensional shapes and 3-dimensional shapes have an area.

Finding of the study suggests that seven of the eight PSSMTs in this study, namely Beng, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, who had selected shapes A, C, D, F, H, I, J, and K that have an area used appropriate mathematical terms to justify their selection of shapes that have an area. This finding is in concurs with the finding of Wun, Lim, and Chew's (2012) study which revealed that 84.78% of the preservice teachers in their study used appropriate mathematical terms to justify their selection of shapes that have an area. However, this finding is in contrast with the finding of Wun and Lim's (2011) study which found that 58% of the preservice special education teachers in their study used appropriate mathematical terms to justify their selection of shapes that have an area.

Finding of the study suggests that all the PSSMTs in this study had taken the effort to justify the selection of shapes that have an area. This finding is in concurs with the finding of Wun, Lim, and Chew's (2012) study which revealed that 95.65% of the preservice teachers in their study had taken the effort to justify the selection of shapes that have an area. This finding is also slightly in concurs with the finding of Wun and Lim's (2011) study which found that 92% of the preservice special education teachers in their study had taken the effort to justify the selection of shapes that have an area.

The implication of this finding is that mathematics educators as well as mathematics teacher educators need to organize teaching and learning activities that provide opportunity for their students and preservice teachers to investigate examples and nonexamples of shapes that have and do not have an area. They included open shapes, 1-dimensional shapes, 2- dimensional shapes, and 3-dimensional shapes because previous studies had shown that some students and preservice teachers encountered difficulty in differentiating between the attributes of perimeter, area, and volume (Baturo & Nason, 1996; Beaumont, Curtis, & Smart, 1986; Ramakrishnan, 1998; Reinke, 1997; Wun & Lim, 2011; Wun, Lim, & Chew, 2012).

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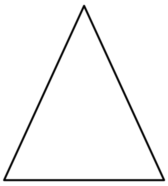
Appendix A

Task 1.2: Notion of area (*luas*)

Tick the shapes that have an area (*luas*).

A



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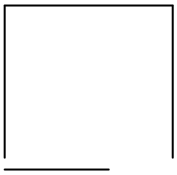

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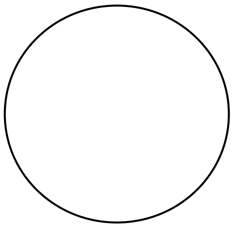

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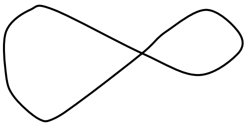

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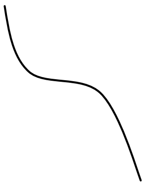

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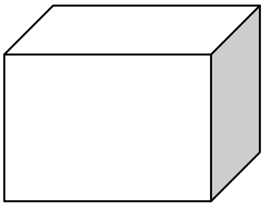

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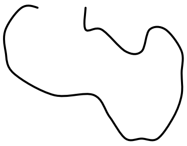

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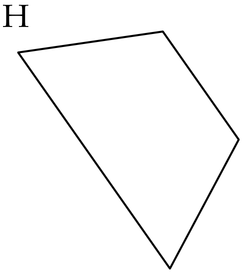

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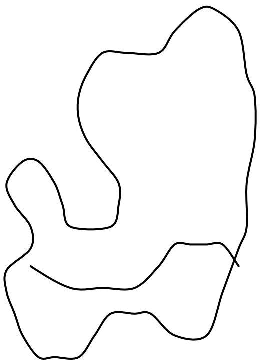

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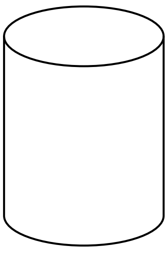

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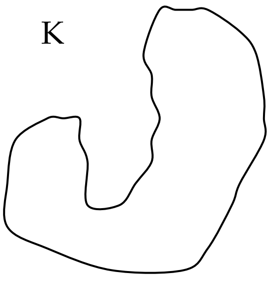

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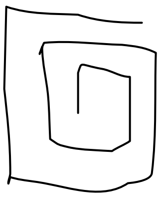

(a) Why did you select this shape/these shapes?

(b) Why didn't you select this shape/these shapes?