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ASSESSMENT OF MATHEMATICAL MODELLING TASKS

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Abstract

Mathematics modelling is now one of the areas that is important in the mathematics education arena. The essence of a modelling problem is constructing a mathematical model for the situation at hand. Assessing mathematical is a field that requires more study. Rubrics that assess the modelling process are a good beginning in finding suitable methods of assessing mathematical modelling. This article suggests an instrument for assessing mathematical modelling tasks. The instrument is a scoring rubric that is based on the modelling process.

Keywords: assessment; mathematical modelling; rubrics; modelling process, modelling tasks

Introduction

In typical mathematics lessons the main focus is always to obtain the correct solution to a problem. Mathematical modelling, however, does not always have a simple and straight forward answer. Because of this, many educators have this question in mind on how do we assess the mathematical modelling process in tasks. Some articles had been written on the assessment of mathematical modelling but the purpose of this article is to provide more insights on mathematical modelling assessment (Goldfinch, 1992; Lingefjard, 2002).

If mathematical modelling is so meaningful and useful then why is the implementation still lagging in the mathematics classroom? One possible reason is that mathematical modelling typically consists of challenging task that requires various competencies. There might be general steps in developing the model but no specific ways in solving them. They require real-world knowledge from domains that may not be as familiar to students and teachers, making their solutions less predictable and complex (Burkhardt, 2004; Ikeda, 2007). In addition modelling tasks are not like the typical "word problems" we have in textbooks that can be solved easily.

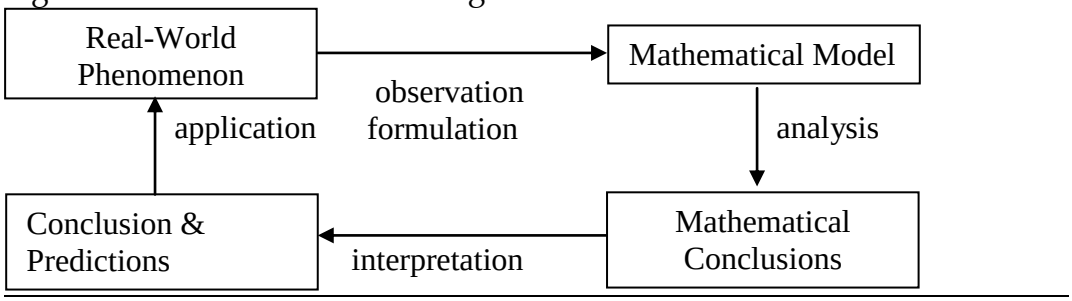
Mathematical Modelling Process

When the word model is mentioned, the first thing that comes to our mind is the image of a physical entity. Most of the time it involves a scaled down version of an object or replica. The term mathematical model generally refers to the mathematical structure that is similar to a real-world problem or phenomenon. This active process

of formulating a model is what we call as mathematical modelling (Swetz & Hartzler, 1991).

Figure 1 shows a simple mathematical modelling process. It is a simple process describing four modelling stages, namely, *Observation, Analysis, Interpretation and Application* (Ang, 1991; Swetz & Hartzler, 1991) although the terms used may differ according to researchers. All modelling process begins with the real world problem that can be formulated into mathematical problems. The mathematical solutions obtained are usually interpreted in the real-world context before it can be accepted.

Figure 1: Mathematical Modelling Process



How to Assess Mathematics Modelling Tasks

Assessment in mathematics education generally refers to the evaluation of the mathematical capability, performance and achievement of students (Niss, 1993). Given the complexity of a mathematical modelling task, however, how does one assess it?

Several studies on assessing mathematical modelling had focused on developing scales that assess the model creation process, assessing specific mathematical ideas used in a model and developing a consistent instrument for different problems (Keck, 1996). Often, educators focus on the products of the modelling process, but this is not sufficient, as the process of mathematical modelling is of importance. For a modelling task, the process could maybe be broken down into two parts, first the general idea followed by the specific ones. An assessment of the modelling tasks are presented in the next section. The instrument is a scoring rubric that is based on the process of mathematical modelling.

Scoring Rubric for Mathematical Modelling Process

To address the competency in the process of mathematical modelling, a rubric was developed to assess the modelling tasks. The outline of the rubric informs the educator of the importance of focusing on the steps while solving the modelling tasks. Assessing the steps of the process of solving the modelling task provides a larger picture of students thinking skill. Even though the final solution is not achieved, nonetheless this rubric might provide some new insight of the process of solving modelling tasks.

This scoring rubric reported a Cronbach alpha's value of 0.72 indicating that the rubric is reliable. Several experts in the field had reviewed this instrument for its content validity and agreed that the items in the scoring rubric reflected the

mathematical modelling process. The rubric below was developed after considering the important checklist that is required in the mathematical modelling process. Some checklist items were adapted from Keck's (1987) scoring scale of mathematical modelling. For each process, a 5-point rating score would be used. The possible rating score is from of 0 to 4 and assigned as follows.

0: Not done
1: Below acceptable
2: Average
3: Good
4: Excellent

Each modelling process is weighted and based on the importance of the process assessed. The items for every process would be given a 0-4 points. For example, in the "Identifying Variables" section, the possible total up points is 12 since each item gets a 0 to 4 score with a weightage of 1. Heavier weights should be given for formulating a model and interpreting the results since these are important steps in the modelling process.

Table 1: Modelling Process Scoring Rubric

Modelling Process	Score	Weight	Total
Identifying Variables 1. States the variables in the model 2. States problem clearly 3. States important features		1	
Formulating a model 1. Creates a model 2. Clearly states all assumptions 3. Describes relationship between variables		3	
Mathematical operations 1. Correct use of mathematics 2. Analyzes relationships between variables 3. Performs operations on the variables relationships		2	
Interpreting the results 1. Reaches solution 2. Interprets solution 3. Evaluates model and solution		3	
Validating the conclusion 1. Revises the model based on the problem		2	

2. Interprets solution based on the revised model 3. Improves the model			
Reporting on conclusions 1. Summarizes the results 2. Reasons about assumptions		1	

Example of Solving a Modelling Task

Task 1: For Birds
Your neighbor, an ornithologist, has to leave for the weekend to do a research study. She has asked you to make sure her birdfeeder always has food in it so that the birds keep coming back throughout the day. Refilling too seldom will cause the birds to look elsewhere for food; refilling too much will scare off the birds. How often should you feed the birds so they keep coming back?

Figure 1: Modelling task (Gould, 2012, p.2)

To solve the birdfeeder problem, one has to go through the mathematical modelling process. Making assumptions is one of the key steps in solving the modelling task. So what do you mean by making assumptions? It generally refers to identifying the important variables in the task. The variables that should be considered include the number of feeding holes in the birdfeeder, how fast does the portion of food empties and if the birds will always be feeding assuming that the bird are not afraid of human replenishing the food supply constantly. Now in the second guiding question, it is given is that the birdfeeder has 4 holes and it takes 45 minutes for the food level to be half full.
The next step is the constructing of the mathematical model. This model can be a graph, equation, table, doing a simulation or a physical model. In this case, the mathematical model would be creating an equation with the defined variables.

Guiding Question:
When you go over first thing in the morning, the birdfeeder — which has 4 holes, one pair near the bottom and another pair about halfway up — is nearly full. You check back 45 minutes later and it's about half full. When do you expect it to empty again?

To construct the equation, define

F = one feeder,
w = the rate at which the feeder empties (feeder/ minute)
t= time taken in minutes

The rate is always constant if $w = \frac{F}{t}$ and this gives $F = wt$. According to the information given, the rate changes at the middle of halfway point. Then we obtain the equation $F = w_1 t_1 + w_2 t_2$.

When the feeder is half-full after 45 minutes, the rate can be found by $\frac{1}{2}F = 45 \times w_1$. Solving the equation gives us the rate, $w_1 = \frac{1}{90}$. Next, we need to get the rate when the feeder is half-full till it is empty which is $w_2 = \frac{1}{2}w_1$. So the rate, $w_2 = \frac{1}{2}w_1 = \left(\frac{1}{2}\right)\left(\frac{1}{90}\right)$ is $w_2 = \frac{1}{180}$.

Then, we substitute all the values that we obtained into the equation $F = w_1 t_1 + w_2 t_2$. We get the value of $t_2 = 90$ minutes. So we would expect the feeder to be empty again in 135 minutes because $t_1 + t_2 = 45 + 90$. The mathematical model which is the equation in this case assisted us in solving the birdfeeder task. After solving this task, one would use the scoring rubric that was developed to assess the mathematical modelling process.

Conclusion

Assessing mathematical modelling is a field that requires more study. Developing more tools and instruments to assess modelling tasks is essential and beneficial for mathematics educators. Rubrics that assess the modelling process would be a good beginning in finding a competence model of assessing mathematical modelling. It is hoped that this instrument would be useful in assessing modelling tasks.

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