

Grinberg's Criterion Applied to Some Non-Planar Graphs

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Abstract

Robertson ([3]) and independently, Bondy ([1]) proved that the generalized Petersen graph $P(n, 2)$ is non-hamiltonian if $n \equiv 5 \pmod{6}$, while Thomason ([5]) proved that it has precisely 3 hamiltonian cycles if $n \equiv 3 \pmod{6}$. The hamiltonian cycles in the remaining generalized Petersen graphs were enumerated by Schwenk ([4]). In this note we give a short unified proof of these results using Grinberg's theorem.

References

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