Assessing The Performance of University Departments Using Data Envelopment Analysis

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Abstract

An efficient use of resources is important in the administration of institutions of higher learning. In this study, data envelopment analysis, DEA, is used to measure the relative efficiency of academic departments of a premier public university, the oldest in the country. Selected input and output indicators are examined and measured utilizing the academic performance of research, teaching and publication. A linear programming based output-oriented DEA models under the assumptions of constant returns to scale, CRS and variable returns to scale, VRS are formulated to compute the relative efficiency of departments of different disciplines. Sources of inefficiency for the inefficient units are computed by utilizing the dual values of their respective peer units. This helps to identify targets and areas where more effort should be devoted to improve the efficiency of the departments.

Keywords: data envelopment analysis, linear programming.
2000 MS Classification: 90C05, 90C32.

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1. Introduction

Exploring knowledge and transmitting knowledge are two main tasks entrusted to a university as an institution of higher learning. The first task is performed and achieved through research activities and results disseminated at meetings such as conferences, symposiums and seminars, and published in academic journals. The second task, transmitting knowledge, is a responsibility of all academic staff and is conducted through face-to-face meeting or teaching (lectures and tutorials) as well as supervision of students, at both undergraduate and postgraduate levels. The relative priority given to the two tasks differentiates between a research university and a teaching university.

In general, there are two types of performance studies conducted on institutions of higher learning. The first focuses on the relative performance of selected universities. The relative efficiency scores are computed, shortcomings assessed and analyzed and comparisons made between universities under investigation. In some cases, rankings are also established. Typical examples include studies on efficiency of Australian universities [1,2], South African universities [3], Canadian universities [4] and performance assessment and efficiency measurement in higher education in Britain [5,6]. The second type of investigation assesses the teaching and research performance of the departments within a single university. Examples include studies conducted by Kao and Hung [7], Koksal and Nalcact [8], and Johnes and Johnes [9]. Beasley [10], on the other hand, compares the performance of the departments of Chemistry and Physics of 52 selected universities in the United Kingdom using an improved model to account for the relative importance of the input and output indicators.

Various methods and techniques, statistical and non-statistical, parametric and non-parametric have been employed in assessing the performance of decision making units, DMUs (such as academic departments and universities). Regression analysis and data envelopment analysis (DEA) are two most popular methodologies cited in the literatures in measuring relative efficiency of DMUs involving multiple inputs and multiple outputs. Several studies credited DEA as a better alternative over regression analysis [11,12], ratio analysis [13], principle component analysis [14], cluster analysis and discriminant analysis [15].

DEA methodology was formally proposed and documented by Charnes, Cooper and Rhodes (abbreviated CCR) in 1978 [16], and is commonly referred to as CCR model. Since then, it has been revised, modified and improved to suit various disciplines and its simplicity and ability to handle multi-input and multi-output production processes without the specification of a production function has made it one the most extensively used performance assessment techniques. In addition to the constant return to scale CCR model, Banker, Charnes and Cooper proposed an alternative variable return to scale BCC model [17]. These models are further classified as either *input-oriented* or *output-oriented*, depending on the type of proportional movement towards the efficient frontier or envelopment surface. An input-oriented model asks the question '*How much can input quantities be proportionately reduced without changing the output quantities produced?*', whereas an output-oriented model addresses the question *'How much can output quantities be proportionately increased using the same amount of input quantities?'*.

The aim of this study is to assess the performance of academic departments and to identify and provide guidance to an inefficient department for its improvement. Altering the levels of inputs such as the number of staff, in short run, is not an easy task. And if we assume a variable return to scale, then an output-oriented BCC model is therefore more appropriate. In addition to providing values of the relative efficiency scores, DEA also makes it possible to identify sources and estimate levels of inefficiency for each inefficient DMUs by utilizing the dual values associated with members of each peer group by constructing a composite DMU which is superior and acts as benchmark to an inefficient DMU under evaluation which will provide guidance in identifying its weak areas that call for improvements.

2. DEA Methodology

DEA formulation is motivated by the classical engineering-science definition of productivity, extended to multiple inputs and outputs. Suppose there are S decision making units (DMUs) to be investigated, each utilizes m inputs to produce n outputs. Further, let DMU_k $(1 \leq k \leq S)$ uses a combination of m inputs, denoted by $X_k = \{X_{k1}, X_{k2}, ..., X_{km}\}$ to produce n outputs, denoted by $Y_k = \{Y_{k1}, Y_{k2}, ..., Y_{kn}\}$. The productivity or relative efficiency, E_k for DMU_k is defined as

$$E_k = \frac{\sum_{j=1}^{n} h_j Y_{kj}}{\sum_{i=1}^{m} c_i X_{ki}}, \quad k = 1, 2, ..., S, \tag{1}$$

where the weights c_i represents the price (i.e the value or shadow cost) of one unit of input X_{ki}, $1 \leq i \leq m$, $\forall\, k = 1, 2, ..., S$, and h_j represents the price (or the value of contribution) of one unit of output Y_{kj}, $1 \leq j \leq n$, $\forall\, k = 1, 2, ..., S$.

Direct application of the above definition is not easy since it requires the determination of the weights to be assigned to each input and output. DEA methodology overcomes this by employing a mathematical programming technique whereby the efficiency ratio defined by equation (1) is further subjected to a number of constraints.

- The efficiency of each DMU must not exceed 100%. Thus $E_k \leq 1.0$, $\forall\, k = 1, 2, ..., S$. If the efficiency score, $E_k = 1.0$, then DMU_k is efficient. Otherwise, if $E_k < 1.0$, then DMU_k is inefficient. This is mathematically equivalent to

$$\sum_{i=1}^{m} c_i X_{ki} - \sum_{j=1}^{n} h_j Y_{kj} \geq 0, \quad k = 1, 2, ..., S. \tag{2}$$

- Further, the costs of all inputs and the prices of all outputs must be strictly positive, resulting in a system of inequalities,

$$c_i \geq \varepsilon > 0, \; i = 1, 2, \dots, m, \text{ and } h_j \geq \varepsilon > 0, \; j = 1, 2, \dots, n,$$

  where ε is an arbitrary small positive number. If $c_i = 0$, DEA is unable to detect and analyze any inefficiency related to the usage of input X_i. Similarly, if $h_j = 0$, DEA is unable to detect and analyze any inefficiency related to the production of output Y_j. Thus, imposing $\varepsilon > 0$ as a requirement to be satisfied by each variable implies that all inputs and outputs are to be regarded as having at least some positive worth, although remains unspecified.

The above conditions lead to the formulation of the fractional programming problem, commonly referred to as CCR-ratio model,

$$\text{maximize}: \quad E_k = \frac{\sum_{j=1}^{n} h_j Y_{kj}}{\sum_{i=1}^{m} c_i X_{ki}}$$

subject to

$$\sum_{i=1}^{m} c_i X_{ki} - \sum_{j=1}^{n} h_j Y_{kj} \geq 0 , \qquad k = 1, 2, \ldots, S .$$

$$c_i \geq \varepsilon > 0 , \quad i = 1, 2, \ldots , m ,$$
$$h_j \geq \varepsilon > 0 , \quad j = 1, 2, \ldots , n .$$

To simplify the computation, we transform the fractional programming problem to a linear programming problem by scaling the input prices so that the total cost of inputs for the DMU under evaluation, say DMU_0 equals 1.0. This calls for an additional constraint $\sum_{i=1}^{m} c_i X_{0i} = 1$. The computation of relative efficiency score for DMU_0 can thus be formulated as

(LP1)

$$\text{maximize}: \quad E_0 = \sum_{j=1}^{n} h_j Y_{0j} \tag{3}$$

subject to

$$\sum_{i=1}^{m} c_i X_{0i} = 1 \tag{4}$$

$$\sum_{i=1}^{m} c_i X_{ki} - \sum_{j=1}^{n} h_j Y_{kj} \geq 0, \qquad k = 1, 2, \ldots , S. \tag{5}$$

$$c_i \geq \varepsilon > 0, \ i = 1, 2, \ldots , m,$$
$$h_j \geq \varepsilon > 0, j = 1, 2, \ldots , n.$$

The above DEA model comprises of $m+n$ decision variables and $S + m + n + 1$ linear constraints, solvable as a linear programming problem. This is termed as the multiplier form of the CCR model under constant returns to scale. If $E_0 = 1$, DMU_0 is said to be CCR-efficient. Otherwise, it is CCR-inefficient. The linear programming problem (LP1) is normally expressed in its dual or envelopment form involving fewer constraints than the primal multiplier form and is generally the most cited and preferred form to solve. This is the BCC version,

(DLP1)

$$\text{minimize} \quad \theta_0 - \varepsilon \left[\sum_{i=1}^{n} s_i^- + \sum_{j=1}^{m} s_j^+ \right]$$

subject to

$$\theta_0 X_{0i} - \sum_{k=1}^{S} X_{ki} Z_k - s_i^- = 0, \quad i = 1,2, \ldots ,n, \tag{6}$$

$$- Y_{0j} + \sum_{k=1}^{S} Y_{kj} Z_k - s_j^+ = 0, \quad j = 1,2, \ldots , m, \tag{7}$$

$$Z_k \geq 0, \quad k = 1,2,\ldots, S,$$
$$s_i^-, s_j^+ \geq 0, \ \forall i, j ,$$
$$\theta_0 \text{ unconstrained.}$$

The dual variable Z_k, $k = 1,2,...,S$, are the shadow prices related to the constraints limiting the efficiency of each DMU to be no greater than 1. The objective here is to find the minimum feasible θ_0 that reduces the inputs X_{0i}, proportionally (or radially) to $\theta_0 X_{0i}$, $\forall i$ while maintaining the output level of DMU_0 no lower than Y_{0j}, $\forall j$. Cooper *et al.* (18) defines the slacks s_i^-, s_j^+, $\forall i, j$ as input excesses and output shortfalls respectively, and are given by

$$s_i^- = \theta_0 X_{0i} - \sum_{k=1}^{S} X_{ki} Z_k, \quad i = 1,2, \dots ,n, \tag{8}$$

$$s_j^+ = \sum_{k=1}^{S} Y_{kj} Z_k - Y_{0j}, \quad j = 1,2, \dots , m, \tag{9}$$

for any feasible solution (θ_0^*, Z_k^*, $k = 1,2,...,S$) relating to DMU_0.

The focus of this study is to seek possible improvements in the levels of outputs, rather than reducing inputs. This calls for the formulation of an equivalent output-oriented DEA model. To do this we express the input-oriented (DLP1) model without the slack variables, that is

(DLP2) minimize θ_0

subject to

$$\theta_0 X_{0i} - \sum_{k=1}^{S} X_{ki} Z_k \geq 0, \quad i = 1,2, \dots ,n, \tag{10}$$

$$-Y_{0j} + \sum_{k=1}^{S} Y_{kj} Z_k \geq 0, \quad j = 1,2, \dots , m, \tag{11}$$

$$Z_k \geq 0, \quad k = 1,2,...,S,$$

$$\theta_0 \text{ unconstrained.}$$

Following [18], we define $\theta_0 = 1/\Omega_0$ and $Z_k = \lambda_k / \Omega_k$, $\Omega_k \neq 0, \forall k$. This transforms (DLP2) into

(DLP3) maximize Ω_0

subject to

$$-X_{0i} + \sum_{k=1}^{S} X_{ki} \lambda_k \leq 0, \quad i = 1,2, \dots ,n, \tag{12}$$

$$-Y_{0j}\Omega_0 + \sum_{k=1}^{S} Y_{kj} \lambda_k \geq 0, \quad j = 1,2, \dots , m, \tag{13}$$

$$\lambda_k \geq 0, \quad k = 1,2,...,S,$$

$$\Omega_0 \text{ unconstrained.}$$

(DLP3) is the output-oriented BCC model under constant returns to scale, CRS. For evaluation under the assumption of variable returns to scale, VRS we impose an additional convexity constraint on λ_k such that

$$\sum_{k=1}^{S} \lambda_k = 1. \tag{14}$$

This results in the formation of a convex hull of intersecting planes which envelope the data points more tightly than the CRS conical hull and thus provides technical efficiency scores which are greater than or equal to those obtained under the assumption of CRS [19]. The difference in the technical efficiency scores under the two assumptions of returns to scale is mainly attributable to scale inefficiency. Thus, scale efficiency, *SE*, can be viewed as the extent to which a DMU can take advantage of returns to scale by altering its size towards the optimal size (defined as the region in which there are constant returns to scale in the relationship between outputs and inputs) and is computed as $SE_0 = \theta_0^{CRS} / \theta_0^{VRS} \leq 1$.

The output-oriented BCC model (DLP3) exhibits some special features:

- The technical efficiency score, $\theta_0 = 1/\Omega_0$, such that $1 \le \Omega_0 < \infty$ since $0 \le \theta_0 \le 1$.
- Proportional improvement in outputs for inefficient DMUs is given by $\Omega_0 - 1$.
- The number of peers among efficient DMUs for an inefficient DMU under evaluation is not more than the number of constraints which corresponds to the total number of inputs and outputs. These peers can be identified from the non-zero λ_k values.
- Each constraint is associated with an input (or output). This provides ease of selecting combinations of input-output mix by enabling/disabling the relevant constraint(s).

In contrast to the input-oriented model, the objective here is to seek maximum Ω_0 that increases Y_{0j} proportionally to $\Omega_0 Y_{0j}$, $\forall j$, while retaining the input level of DMU$_0$ no greater than X_{0i}. $\forall i$. Improvement or movement towards efficient frontier by inefficient DMUs can be identified by inspecting the system of equations (12) and (13). Define the slacks t_i^-, t_j^+, $\forall i, j$ by

$$\sum_{k=1}^{S} X_{ki}\lambda_k + t_i^- = X_{0i}, \quad i = 1,2, \ldots ,n, \tag{14}$$

and

$$\sum_{k=1}^{S} Y_{kj}\lambda_k - t_j^+ = Y_{oj}\Omega_0, \quad j = 1,2, \ldots , m. \tag{15}$$

For an inefficient DMU$_0$, say, the projected output on the efficient frontier is as dictated by its peers (identified from $\lambda_k \ne 0, \forall k$) and given by $\sum_{k=1}^{S} Y_{kj}\lambda_k$, $j = 1,2,\ldots,m$. This can be achieved by proportional improvements of $(\Omega_0 - 1)$ in all outputs plus additional amount (termed as slack movements) of t_j^+ in output Y_{0j} whenever $t_j^+ \ne 0$. On the input side, equation (14) suggests that the level of input X_{0i}, $\forall i$ can further be reduced by an amount of t_i^- whenever $t_i^- \ne 0$ to those dictated by the peers, i.e $\sum_{k=1}^{S} X_{ki}\lambda_k$. Thus, $(\Omega_0 - 1)Y_{0j} + t_j^+$ is a measure of *under-achievement* of output Y_{0j}, $j = 1,2,\ldots,m$, experienced by DMU$_0$, while t_i^- reflects the *over-utilization* of input X_{0i}, $\forall i$. The projected position on (and the movement to) the efficient frontier can be expressed as

$$X_{0i}^{\wedge} = \sum_{k=1}^{S} X_{ki}\lambda_k^* = X_{0i} - t_i^{-*}, \; i = 1,2, \ldots ,n, \tag{16}$$

and

$$Y_{0j}^{\wedge} = \sum_{k=1}^{S} Y_{kj}\lambda_k^* = Y_{0j}\Omega_0^* + t_j^{+*}, \quad j = 1,2, \ldots, m. \tag{17}$$

where $(X_{0i}^{\wedge}, Y_{oj}^{\wedge}, \forall i, j)$ is the position of the composite virtual efficient DMU on the frontier, and $(\Omega_0^*, t_i^{-*}, t_j^{+*}, \lambda_k^*)$ is the optimal solution of (DLP3) for the decision making unit under evaluation, DMU$_0$.

3. Selections of input-output indicators and DMUs

Teaching and research have been universally accepted as two major tasks of a university but they are not easy to quantify or measure. However, there appear to be no general agreement among researchers with regard to the choice of suitable input and output indicators for studies in academic institutions or departments. Thus, indicators have to be carefully identified and selected to represent or reflect the sources of input and achievements of these activities. The selection is usually constrained by the availability and accessibility of data. Since the focus of the study is to access the performance of efforts devoted to teaching and research, only measurable indicators are selected as the input and output variables.

Input variables represent the factors (in quantified form) utilized by the department in the delivery of its services. Two input indicators are selected for the study. These are

- *Academic staff*, as measured by the total number of full-time teaching and research personnel, which reflects the staff input to the process of teaching and research of an institution of higher learning. This is synonym to labour input in the normal production activity.
- *Operating expenses* allocated to the department by the university which include expenses for the procurement and maintenance of equipments and facilities, stationary, travels and other types of internal expenditures. This is synonym to capital expenditure in the normal production process.

Output variables represent and measure the achievements of a department in performing its tasks. A broad range of selections are considered in the literatures. Three output indicators are shortlisted for the study. These are

- *Aggregate work load* which is computed as the number of credit-hours of a course multiplied by the number of students taking that course summed over all the courses taught by the department divided by the number of full-time academic staff [7]. This indicator reflects the teaching activity allocated to each staff of a department, and is regarded as an improvement to the 'number of students registered'.
- *Research income*, computed as the total research fund and grants acquired by the staff member of the department. This indicator is included in [6] to reflect both the quality and quantity of research output, while [10] regarded it as an input as well as a proxy for output. In most cases, the total amount of grants allocated/acquired by an institution is an important indicator in evaluating its research capability.
- *Number of weighted postgraduate theses* produced by the departments as an alternative to the most widely accepted indicator, *the number of publications* of staff member in selected cited journals. Our choice is hindered by the non-availability of reliable data on the number of publications before 2007. In addition, the input data on operating expenses were made available only up to 2006. We compute the weighted sum of the number of theses as M.Sc (1 point) and Ph.D (2 points).

In short, two input and three output indicators are used to assess the performance and achievement of a department in teaching and research. In terms of variables, these are

X_1 : number of full-time academic staff of a department,
X_2 : annual operating expenses allocated to the department,
Y_1 : aggregate workload per academic staff,
Y_2 : total annual research income, and
Y_3 : total weighted number of theses produced.

Selection of DMUs.
We selected 17 academic decision making units from four faculties for our study. This is consistent with most practices where the number of sample size should be at least three times larger than the sum of inputs and outputs in order to discriminate effectively between efficient and inefficient DMUs [2]. The first faculty, designated as DMU01 comprises of four academic departments, jointly offering a single undergraduate degree and four postgraduate programs. It is currently one of the smallest faculty with 761 undergraduate students, 348 postgraduate students and 67 academic staff. The second faculty comprises of six academic departments, designated DMU02, DMU03, DMU04, DMU05, DMU06 and DMU07, offering professional degrees. Each department is entrusted with separate undergraduate degrees, a total of 11 different degrees with an additional of 11 programs at the postgraduate level. The departments share a minimum (less than 5%) number of common courses. The third faculty is among the oldest and most accomplished in the University. It is currently the largest faculty with nearly 3000 undergraduate students, 900 postgraduate students, 220 academic staff and 250 support staff. This faculty comprises of six departments, designated as DMU08, DMU09, DMU10, DMU11, DMU12 and DMU13, offering a total of 24 undergraduate degrees and 11 postgraduate programs. The fourth faculty is the youngest with its first enrollment for 1990/1991 academic session of only 50 students. It now comprises of four departments, designated DMU14, DMU15, DMU16 and DMU17, offering two undergraduate degrees with seven areas of specializations and five postgraduate programs. More than half of the undergraduate courses are common to all departments.

4. DEA Results and Interpretations

Descriptive statistics for all input and output indicators are displayed in Table 1 for the three years studied (2004–2006). It is clear from the table that the number of academic staff (variable $X1$) does not exhibit significant annual variation as opposed to input variable $X2$ (operating expenses) which indicates an average increase of about 20% from RM2.613 million in 2004 t0 RM3.138 million in 2006. On the output side, the average teaching load per academic staff shows a drop of about 14.5% from 530 unit in 2004 to 453 unit in 2006. This is equivalent to saying a reduction of 8 undergraduate students from 53 to 45 for a 10 credit hour teaching load. The average research income fluctuates from RM133,093 (in 2004) to RM113,092 (in 2005) to RM123,447 (in 2006). However, the maximum recorded shows a decline trend from RM665,821 to RM525,000. The weighted number of theses produced at postgraduate level on average doubles during the years under study from a maximum of 44 (in 2004) to 79 (in 2006).

Table 1 Descriptive Statistics

Year	Variable	Mean	Maximum	Minimum	Std. Dev.
2004	*X1:*	26	66	11	15.38
	X2:	2613053	7142000	184600	2004560
	Y1:	530.14	798.05	167.71	188.98
	Y2:	133093	665821	0	171578
	Y3:	8.18	44	0	10.30
2005	*X1:*	26	67	8	17.69
	X2:	3056982	9999300	172200	2479490
	Y1:	529.71	1155.25	268.54	213.36
	Y2:	113092	540900	0	132907
	Y3:	13.47	67	1	15.76
2006	*X1:*	26.706	72	10	17.61
	X2:	3137759	10969700	311000	2689869
	Y1:	453.00	754.58	233.50	173.20
	Y2:	123447	525000	0	172013
	Y3:	16.82	79	1	132.07

Next, we use a linear programming software, LINDO to solve the output-oriented DEA model, (DLP3), under the assumptions of CRS and VRS for the three separate years. Each DMU is assessed relative to the performance of the other 16 DMUs in each period. This amounts to running the program 102 times. The relative technical efficiency scores are summarized in Table 2. Results for the first two consecutive years, 2004 and 2005 reveal that the technical efficiency scores produced under CRS and VRS do not exhibit large deviations except for two DMUs in 2004 (DMU02 and DMU03) and three DMUs in 2005 (DMU01, DMU12 and DMU16). The mean absolute deviations, MAD for 2004 and 2005 are 6.33% and 5.93 % respectively. In 2006, more than half of the DMUs produced significantly different technical efficiency scores under CRS and VRS with a higher MAD value of 16.21%. Thus, assuming CRS may be appropriate for 2004 and 2005 but not for 2006 which indicates the presence of VRS and hence scale inefficiencies. The scale efficiency scores, SE are also computed for each DMU and displayed in Table 2. Since the results are all *relative,* they provide information on how each individual department performed in comparison to other departments in the year under consideration.

Nine DMUs remain 100% efficient under VRS for the three consecutive years. These are DMU01 (a faculty), DMU03 and DMU07 (from the second faculty), DMU08, DMU09, DMU10 and DMU13 (from the oldest faculty) and DMU14 and DMU16 (from the youngest faculty). However, under the assumption of CRS, only five DMUs maintained their 100% efficiency scores. A closer examination reveals that only three DMUs show improvements in TE_{VRS} scores while five DMUs experienced drops in their TE_{VRS} scores from 2004 to 2006. The mean values for TE_{CRS}, TE_{VRS} and SE declined from 86.86% in 2004 to 70.99% in 2006, 93.19% in 2004 to 87.20% in 2006 and 92.71% in 2004 to 80.48% in 2006 respectively. Further, four of the ten DMUs with TE_{VRS} scores equal to one in 2006 attained relatively low TE_{CRS} scores. These are DMU01, DMU03, DMU08 and DMU16. This implies that these DMUs are able to transform their set of inputs into a set of outputs efficiently, but the technical efficiencies (under CRS) are low due to their disadvantage size (DMU01 is the smallest faculty and DMU08 is one of the largest departments). The other six DMUs are all scale efficient where their SE scores equal to one. They are also said to have achieve the *most productive scale size, mpss.*

The last column in Table 2 presents the type of returns to scale exhibited by each DMU in 2006. This is conducted by solving the output-oriented (DLP3) model under the assumption of *non-increasing returns to scale, NIR* by imposing the constraint $\sum_{k=1}^{S} \lambda_k \leq 1$. If the technical efficiency score obtained is equal to the

Table 2. Relative efficiency estimates, 2004 - 2006

DMU	2004			2005			2006			RTS
	TE_{CRS}	TE_{VRS}	SE	TE_{CRS}	TE_{VRS}	SE	TE_{CRS}	TE_{VRS}	SE	
DMU01	0.87365	1.00000	0.87365	0.72016	1.00000	0.72016	0.55187	1.00000	0.55187	drs
DMU02	0.37726	0.70919	0.53195	0.78567	0.85502	0.91889	0.50674	0.75904	0.66761	drs
DMU03	0.59753	1.00000	0.59753	1.00000	1.00000	1.00000	0.68399	1.00000	0.68399	irs
DMU04	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.59750	0.81958	0.72903	drs
DMU05	0.53468	0.54275	0.98513	1.00000	1.00000	1.00000	0.67323	0.68151	0.98785	drs
DMU06	0.95299	1.00000	0.95299	1.00000	1.00000	1.00000	0.47433	0.75899	0.62494	drs
DMU07	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	mpss
DMU08	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.46418	1.00000	0.46418	drs
DMU09	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	mpss
DMU10	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	mpss
DMU11	0.98586	1.00000	0.98586	0.67962	0.68626	0.99033	0.38470	0.50485	0.76201	drs
DMU12	0.73793	0.75863	0.97272	0.41277	0.60545	0.68176	1.00000	1.00000	1.00000	mpss
DMU13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	mpss
DMU14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	mpss
DMU15	0.92037	0.97544	0.94354	0.51304	0.53279	0.96292	0.54968	0.72164	0.76171	irs
DMU16	1.00000	1.00000	1.00000	0.73044	1.00000	0.73044	0.81752	1.00000	0.81752	irs
DMU17	0.78620	0.85680	0.91760	1.00000	1.00000	1.00000	0.36434	0.57787	0.63049	drs
mean	0.86862	0.93193	0.92712	0.87304	0.92232	0.94144	0.70989	0.87197	0.80478	

TE_{CRS} is the technical efficiency estimate under the assumption of constant returns to scale

TE_{VRS} is the technical efficiency estimate under the assumption of variable returns to scale

SE is the measure of scale efficiency.

RTS is the returns to scale, with drs and irs denoting decreasing and increasing to scale respectively, and mpss denotes the most productive scale size.

Table 3. Results for inefficient DMUs under VRS (year 2006)

DMUs	Variable	Original value	Radial movement	Slack movement	Projected value
DMU02	X1	25	0.00	0.00	25
Ω_2=1.31745	X2	2923900	0.00	0.00	2923900
TE_{VRS}=0.75904	Y1	499.44	158.55	0.00	657.99
SE = 0.66761	Y2	60000	19047	0.00	79047
	Y3	13	4.13	0.00	17.13
List of peers: DMU01, DMU08,DMU09,DMU12,DMU13					
DMU04	X1	22	0.00	0.00	22
Ω_4=1.22014	X2	3389400	0.00	374545	3014855
TE_{VRS}=0.81958	Y1	570.45	125.58	0.00	696.03
SE = 0.72903	Y2	90000	19812.60	0.00	109812.60
	Y3	9	1.98	0.00	10.98
List of peers: DMU01, DMU08,DMU12,DMU13					
DMU05	X1	20	0.00	0.00	20
Ω_5=1.46733	X2	2234600	0.00	0.00	2234600
TE_{VRS}=0.68151	Y1	351.5	164.27	0.00	515.77
SE = 0.98785	Y2	170000	79446.10	0.00	249446.10
	Y3	8	3.74	0.00	11.74
List of peers: DMU07,DMU09,DMU10,DMU12,DMU14					
DMU06	X1	25	0.00	0.00	25
Ω_6=1.31754	X2	2820500	0.00	0.00	2820500
TE_{VRS}=0.75899	Y1	530.24	168.37	0.00	698.61
SE = 0.62494	Y2	60000	19052.40	0.00	79052.40
	Y3	9	2.86	0.00	11.86
List of peers: DMU01, DMU08,DMU09,DMU12,DMU13					
DMU11	X1	17	0.00	0.00	17
Ω_{11}=1.98079	X2	2307500	0.00	494225	1813275
TE_{VRS}=0.50485	Y1	348.94	342.24	0.00	691.18
SE = 0.76201	Y2	0	0.00	2876.6	2876.60
	Y3	5	4.90	0.00	9.90
List of peers: DMU01, DMU13, DMU14					
DMU15	X1	12	0.00	0.00	12
Ω_{15}=1.38573	X2	1156500	0.00	0.00	1156500
TE_{VRS}=0.72164	Y1	285.08	109.96	0.00	395.04
SE = 0.76171	Y2	15000	5785.95	0.00	20785.95
	Y3	5	1.93	0.00	6.93
List of peers: DMU03, DMU07,DMU13,DMU14,DMU16					
DMU17	X1	26	0.00	0.00	26
Ω_{17}=1.73049	X2	4060800	0.00	1456574	2604226
TE_{VRS}=0.57787	Y1	288.54	210.78	0.00	499.32
SE = 0.63049	Y2	0.00	0.00	6835.39	6835.39
	Y3	17	12.42	0.00	29.42
List of peers: DMU01,DMU13,DMU14					

TE_{VRS} score, then the corresponding DMU is said to be operating under *decreasing returns to scale, drs*. Otherwise, it is said to be operating under *increasing returns to scale, irs*. Thus, for the year 2006, six departments were operating at most productive scale size, eight exhibiting decreasing returns to scale and three exhibiting increasing returns to scale. Of the seven technically inefficient DMUs, only DMU15 exhibits increasing returns to scale. The remaining six exhibit decreasing returns to scale.

Identification of sources of inefficiency
In addition to providing scores for the relative technical efficiency, DEA also identifies sources of inefficiency inherent in the inefficient DMUs and projects targets or levels to be adopted by these DMUs if they are to be on the efficient frontier. These results for the seven inefficient DMUs in 2006 under VRS are summarized in Table 3. We will analyze selected DMUs to highlight the concept involved.

a) DMUs with zero slacks

These are DMU02, DMU05, DMU06 and DMU15. Their projected values are fully dictated by their peers and given by systems of equations (16) and (17) with $t_i^- = 0,\ t_j^+ = 0,\ \forall i, j$. Thus, for DMU02, for example, we have $\Omega_2^* = 1.31745$, giving

$$X_{2i}^{\wedge} = X_{2i},\ i = 1,2,\ldots,n,$$

$$Y_{2j}^{\wedge} = 1.31745Y_{2j} = Y_{2j} + 0.31745Y_{2j},\ j = 1,2,\ldots,m.$$

This means all outputs are to be proportionally increased by 31.745% in all directions. These incremental values are associated with the radial movements and are given under the fourth column in Table 3. The projected values are the sum of the original values and their respective radial movements. These are recorded under the last column and represent the position of an *efficient virtual composite DMU (of peers)* on the efficient frontier which benchmarks the position of the inefficient DMU. Similar interpretation applies to DMU05, DMU06 and DMU15.

b) DMUs with non-zero slacks

Next, we turn to DMU04. The result indicates the presence of a non-zero variable slack, $t_2^- = 374545$, associated with input X_2, operating expenses. The position on the frontier is achieved by a radial movement of 22.0% of all outputs, followed by a reduction of RM374,545 in input X_2. Thus an output-oriented model also identifies over-utilized inputs as given by their excessive slacks. DMU11 and DMU17 depict similar results with non-zero slacks for input X_2 and output Y_2. Both DMUs were using surplus operating expenses and calls for a reduction of about 21.4% and 35.9% respectively. A movement in all outputs alone is not sufficient to project the DMUs onto the efficient frontier. A slack movement of RM2876.60 (for DMU11) and RM6835.39 (for DMU17) for output Y_2 is required for the two DMUs to match their virtual composite DMUs on the frontier. We can represent the results for DMU11 in terms of equations (16) and (17) as follows,

$$\begin{cases} X_{11(1)}^{\wedge} = X_{11(1)} - t_1^- = 17 - 0 = 17, \\ X_{11(2)}^{\wedge} = X_{11(2)} - t_2^- = 2307500 - 494225 = 1813275, \\ Y_{11(1)}^{\wedge} = 1.98Y_{11(1)} + t_1^+ = (348.94 + 342.24) + 0 = 691.18, \\ Y_{11(2)}^{\wedge} = 1.98Y_{11(2)} + t_2^+ = (0 + 0) + 2876.60 = 2876.60, \\ Y_{11(3)}^{\wedge} = 1.98Y_{11(3)} + t_3^+ = (5 + 4.9) + 0 = 9.90. \end{cases}$$

A similar representation can be deduced for DMU17.

5. Conclusion

Academic institutions are important component of human capital formation, as well as one the major expenditure components for taxpayers [1]. Thus, performance assessment (in terms of efficiency) of these units is an important public policy issue. In this paper, we report the assessment of the performance of seventeen selected academic units of a premier public university in Malaysia for three consecutive years, 2004-2006 using an output-oriented DEA model under the assumptions of CRS and VRS. The technical and scale efficiency estimates suggest that the selected units were operating at a fairly high level of efficiency relative to each other. However, there exist rooms for improvement is some units.

The paper also highlights how DEA can be used to estimate and identify inefficiencies and their sources, over-utilization of input resources and shortfalls in output levels. For inefficient units, DEA also identifies the associated efficient virtual composite units on the frontier comprising of relevant group of peers of efficient units and the directions to these projected composite units. This information can aid public policy-makers and university executives in allocating scarce resources more efficiently and identifying directions for improvement.

Lastly, it is acknowledged that the study is by no means complete. Due to limited space and time, many important aspects of DEA have not been addressed. Some of these topics include the multiplier or weight restrictions such as the imposition of assurance regions (AR), issues of congestion, the restriction of integer-value variables, general multiple criteria decision making such as GoDEA and integrated analytic hierarchy process (AHP), dynamic changes in efficiency over time involving technological change and frontier shift (a study in Malmquist's total factor productivity), and random variable data chance constrained programming for the formulation of probability-based stochastic DEA model. These topics are receiving significant attention in literatures and provide directions and avenues for future research.

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