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COMPONENT COMMONALITY IN MULTISTAGE PRODUCTION SYSTEM: MODELS WITH CAPACITY CONSTRAINTS

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ABSTRACT

Common components are used extensively for production postponement, reduction of proliferated product lines cost, reduction of the cost of safety stock, increase productivity, improve flexibility, expediting new product development and so on. The authors consider a multistage assemble-to-order system with two products have uniformly distributed demand, one common component and product-specific components. We develop optimization models for minimizing the level of inventory of the components and allocated to products to meet capacity limitation. A numerical example is used to verify and compare the models with similar models.

Keywords: *Multistage, commonality and models*

INTRODUCTION

Component commonality refers to a manufacturing environment where two or more products use the same components in their assembly. Commonality is an integral element of the increasingly popular assemble-to-order strategy that inventories certain critical components –typically, with long lead time and expensive – in a generic form [1]. Commonality is an approach in manufacturing, production and inventory management system where different components replace by common component(s) or same components are used for multiple products and thereby simplifies the management and control of resources and ease the analysis and improvement of existing products/processes or development of new products/processes at an optimize costs. For more details on commonality the authors would like to humbly refer the readers to Wazed et al. [2].

Commonality substantially lowers the costs of proliferated product lines, mitigate the effects of product proliferation on product and process complexity [3]; reduces the cost of safety stock, decrease the set-up time, increase productivity, and improve flexibility [4]; reduces the required number of order (or setups) [1, 5]; reduces risk-pooling and lead time uncertainty, improve the economy of scale, simplify planning, schedule and control, streamline and speed up product development process [6]; facilitates quality improvement, enhance supplier relationship and reduce product development time [1] etc. The benefits and limitations of commonality are summarized in Wazed et al. [2].

Multi-stage production is a system which transforms or transfer inventories through a set of connected stages to produce finished goods. This system may contain stages which represent the delivery or transformation of raw materials, the transfer of work-in-process between production facilities, the assembly of component parts, or the distribution of finished goods. Normally, the manufacturing process of a product and its associated multiple versions involve in multi-stage production system, each requiring different input parts and subassemblies. Increasing a level of part commonality at the early stages of the assembly process can be considered as postponing the differentiation of products until after these early stages. MRP and MRP II are used throughout industry to determine production schedules in multi-stage manufacturing systems. The basic idea (see Orlicky [7] or McClain *et al.* [8]) is that a production schedule of a finished item translates into known quantity and timing needs for components, based on bill-of-material and lead-time information. Several difficulties arise in practice, including unpredictable lead times, facilities with limited capacity, unpredictable external demand for components, random yields, defective items, and changes in the final product schedule.

This paper presents mathematical models for dealing with capacity constraints under predictable production lead times, and demonstrates the effects of commonality on the parameters. In this article the authors focus on multistage production environment under commonality. We pay particular attention to the link between incorporation of commonality in multistage, multi-product and multi-period production system and inventory level.

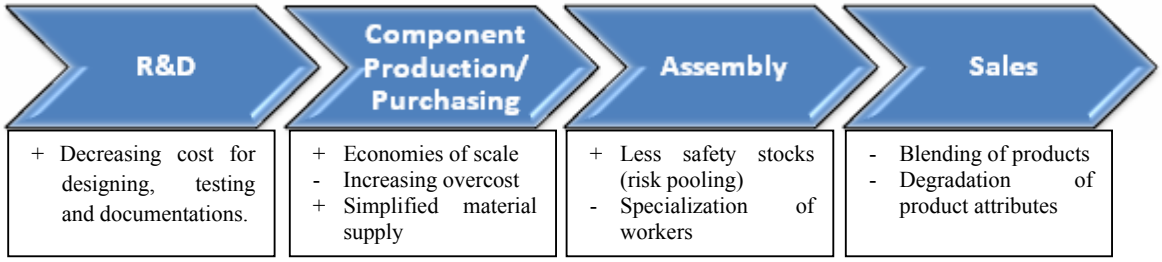

Figure 1: Impact of increasing component commonality along the supply chain

MULTISTAGE PRODUCTION MODELS

Multi-stage production systems have been analyzed by hundreds of authors, and we will not attempt to cover all of the work in this area. Figure 1 depicts just a brief extract of the cost possessions of common parts discussed in literature. More exhaustive reviews are provided by Kamalini [9], Labro [10] as well as Wazed *et al*. [11]. With regard to the variety of different relationships between component commonality and supply chain operations it is not amazing that a massive body of literature has accumulated. Three major rivulets of research can be acknowledged: (i) inventory and operations related commonality research, (ii) R&D and engineering related commonality research and (iii) marketing related commonality research. Any of these brooks covers a definite excerpt of the overall problem and any stream by itself contains a multitude of diverse research papers investigating specific component commonality problems. Consequently, plenty different modeling approaches have been introduced, none of which being dedicated to the commonality problem treated in this article. None of the models, reported so far, covers the effects of commonality in a multistage, multi-product and multi-period production/manufacturing environment.

The Models

We consider a simple model of a product family consisting of three end products (A, B and C). The products are produced from 14 basic raw materials (A: AA, AD, AE & AF; B: BD, BG, BI, BJ & BK; and C: CA, CD, CE, CG & CH). The Figure 2 shows the products structure for the basic form of our models, in which each product has their own unique components. Demand and lead time distribution is certainly known for all products and components. General purpose (i.e. universal) work centers of six with limited capacity are considered. For the commonality models, the unique components (AF, BJ & CG) are replaced with a common component (ABC), having no-negative additional cost (co) and inventory level (IC).

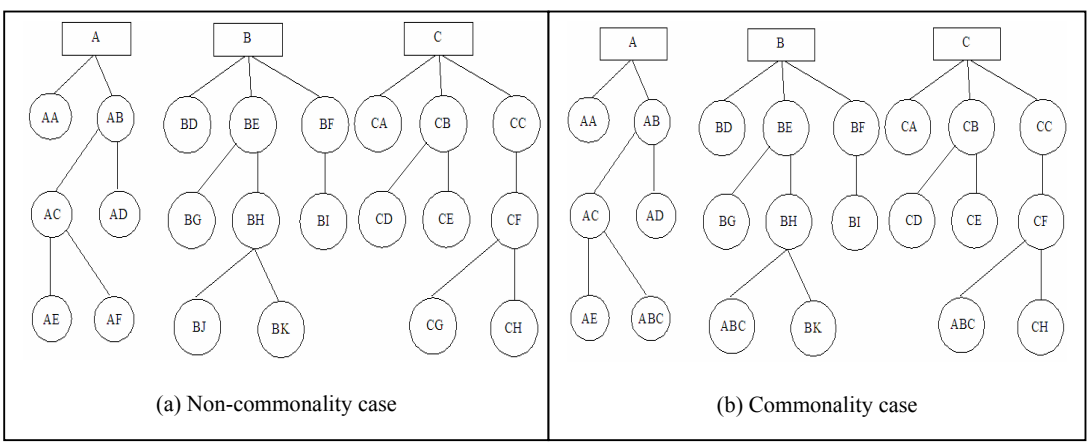

Figure 2: Two product structures of a product family

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Variables and parameters

I_{it}	Inventory of the product *i* at the end of period t
x_{it}	Unit of end product/component i produce in time t
D_{it}	Demand of product/component i at time t
R_{ij}	Amount of component i needed to produce product j
δ_{it}	=1, if any product/component i started at time t; 0 otherwise.
$LS(i)$	Minimum lot size for product/component i
M	A large number
$U(i,k)$	Fraction of resources k needed to make one unit of product/component i
$ST(i,k)$	Fraction of resources k used to setup for product/component i
γ_{ikt}	=1, if component/product i will be the last product produced on resource k in time bucket t-1 and the first produced in time bucket t; 0 otherwise.
v_i	Production/processing cost of product/component i
q_i	Inventory holding cost of product/component i per unit time
$ENDP$	Set of all the end items (items with external demand)
co_i	Additional processing cost of common component i

The formulation of the problem is follows:

Non-commonality cases:

$$Minimize: \quad z = \sum_{i=1}^{I}\sum_{t=1}^{T}\left(v_i x_{it} + q_i I_{it}\right) + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} c_i\left(\delta_{it} - \gamma_{ikt}\right) \quad \text{.....(1)}$$

Subject to:

$$I_{it-1} + \sum_{\tau=1}^{t-LT(i)} x_{i\tau} - I_{it} \geq D_{it} \quad i = 1,..,ENDP; \quad t = 1,..,T \quad \text{.....(2)}$$

$$I_{it-1} + \sum_{\tau=1}^{t-LT(i)} x_{i\tau} - I_{it} \geq \sum_{j=1}^{N} R_{ij} x_{jt} \quad i = 1,..,N \setminus ENDP; \quad t = 1,..,T \quad \text{.....(3)}$$

$$x_{it} - \delta_{it} LS(i) \geq 0 \quad i = 1,..,N; \quad t = 1,..,T \quad \text{.....(4)}$$

$$x_{it} \leq M\delta_{it} \quad i = 1,..,N; \quad t = 1,..,T \quad \text{.....(5)}$$

$$x_{it} \geq 0; \quad \delta_{it} \in \{0,1\} \quad i = 1,..,N; \quad t = 1,..,T \quad \text{.....(6)}$$

Capacity Constraints:

$$\sum_{i=1}^{N}\left(U(i,k)x_{it} + ST(i,k)(\delta_{it} - \gamma_{ikt})\right) \leq 1 \quad k = 1,.....,K; \quad t = 1,.....,T \quad \text{.....(7)}$$

$$\delta_{it-1} + \delta_{it} \geq 2\gamma_{ikt} \quad i = 1,.....,N; \quad k = 1,.....,K; \quad t = 1,.....,T \quad \text{.....(8)}$$

$$\gamma_{ikt} \leq MU(i,k) \quad i = 1,.....,N; \quad k = 1,.....,K; \quad t = 1,.....,T \quad \text{.....(9)}$$

$$\sum_{i=1}^{N}\gamma_{ikt} \leq 1 \quad k = 1,.....,K; \quad t = 1,.....,T \quad \text{.....(10)}$$

The objective function includes the variable cost of product, inventory carrying costs and the setup costs only. Constraints (2) and (3) insures that production will occur at least LT (lead time) time periods before needed for end products and components respectively. This additional waiting time will be based on an actual production bottleneck, rather than on past experience and guesswork as is the usual situation. This is the reason why lead time should under ideal circumstances. Constraint (4) assures to produce/process a quantity of lot size in single run. If there is any positive production/processing, setup variables are set to be 1 [constraint (5)]. Constraint (6) provides the integrality and non-negativity requirements. The capacity of each facility in each time period will not be exceeded except by use of overtime is insured by constraint (7). The constraints labeled (8) allow γ to

be one for component/product i on resource k only if there is production of component/product i in both periods. Constraints (9) ensure that we only set γ to 1 for component/product i that is/are to be routed to resource k, which is done mainly to avoid spurious values of γ that can be confusing when reading the solution. Constraints (10) ensure that at most one product can span the time boundary on a specific resource k. In order to have constraints (8) make sense, we must define δ_{i0} as data. This is very general model for multistage, multi-product and multi-period production systems.

Commonality case:

Objective function:

$$Minimize: \quad z = \sum_{i=1}^{I}\sum_{t=1}^{T}\left((v_i + co_i)x_{it} + q_i(I_{it} + IC_{it})\right) + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} c_i\left(\delta_{it} - \gamma_{ikt}\right) \quad \text{.....(11)}$$

Subject to:

$$I_{it-1} + \sum_{\tau=1}^{t-LT(i)} x_{i\tau} - I_{it} \geq D_{it} \quad i = 1,..,ENDP; \quad t = 1,..,T \quad \text{.....(12)}$$

$$I_{it-1} + IC_{it-1} + \sum_{\tau=1}^{t-LT(i)} x_{i\tau} - I_{it} - IC_{it} \geq \sum_{j=1}^{N}\left(R_{ij}x_{jt} + R_{cj}x_{jt}\right) \quad i = 1,..,N \setminus ENDP; \quad t = 1,..,T \quad \text{.....(13)}$$

$$IC_{it-1} - IC_{it} \geq \sum_{j=1}^{N} R_{cj}x_{jt} \quad i = 1,..,N \setminus ENDP; \quad t = 1,..,T \quad \text{.....(14)}$$

$$x_{it} - \delta_{it}LS(i) \geq 0 \quad i = 1,..,N; \quad t = 1,..,T \quad \text{.....(15)}$$

$$x_{it} \leq M\delta_{it} \quad i = 1,..,N; \quad t = 1,..,T \quad \text{.....(16)}$$

$$x_{it} \geq 0; \quad \delta_{it} \in \{0,1\} \quad i = 1,..,N; \quad t = 1,..,T \quad \text{.....(17)}$$

Capacity Constraints for commonality case are same like the non-commonality models.

Numerical Example

As mentioned earlier, Fig. 2 is used to analysis the outcomes of our proposed models. The product family has three end products, namely A, B and C. The raw materials and subcomponent for the products are shown in Fig. 2. The relevant data like lead time, component requirements, and lot size are shown in the Table 1. The precedence relation and requirement of components are noted in Table 2. The cost parameters are considered as constant as our intension is to analyze the inventory level for common components and for the components it replaced under the capacity limit of the general purpose work centers. The machine setup time and the processing time for the products/components are random variable and have chosen from ranges 25-40 and 5-25 minutes respectively. We have considered the lead time of 4 (maximum of the components it replaced) for common component and the lot size of 20 (same with non-commonality). All other variables and parameters are kept unchanged. The disparity of inventory level at different periods for commonality and non commonality cases are shown in Figure 3. It is pellucid that the inventory level is higher for common component than the components it replaced, but less than the sum the components.

Table 1: Initial information of the production system

Products/Parts Name	Lot Size	Lead time	Initial inventory	Demand range of end products
A	10	1	0	10-20
AA	20	2	10	
AB	20	1	10	
AC	20	1	10	
AD	20	3	10	

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AE	20	1	10	
AF	20	1	10	
B	10	2	0	15-30
BD	20	2	10	
BE	20	1	10	
BF	20	3	10	
BG	20	2	10	
BH	20	1	10	
BI	20	1	10	
BJ	20	2	10	
BK	20	3	10	
C	20	2	0	10-20
CA	20	1	10	
CB	20	2	10	
CC	20	2	10	
CD	20	2	10	
CE	20	2	10	
CF	20	1	10	
CG	20	4	10	
CH	20	2	10	

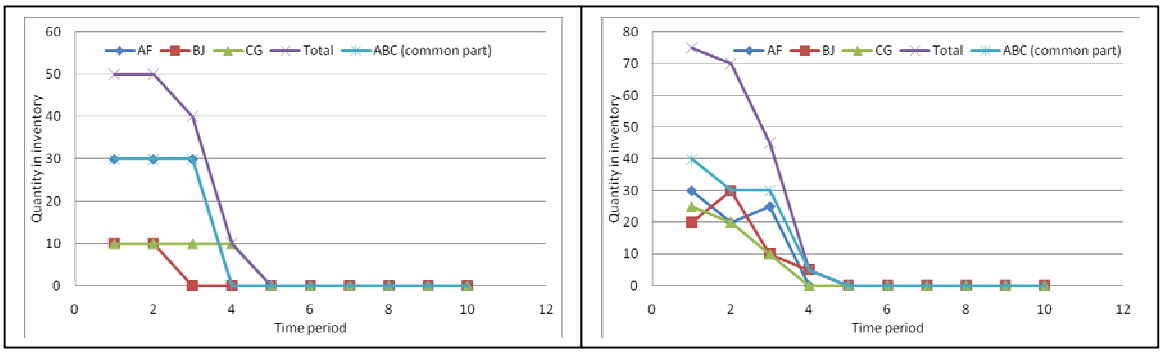

Figure 3: Level of inventory [Demand: A=10, B=15 and C=10 (left); A=15, B=10 and C=15 (right)]

Table 2: Precedence and requirement list

Non-Commonality case			Commonality case		
Product/parts	Immediate successor	quantity	Product/parts	Immediate successor	quantity
A	AA	4	A	AA	4
A	AB	3	A	AB	3
AB	AC	3	AB	AC	3
AB	AD	2	AB	AD	2
AC	AE	5	AC	AE	5
AC	AF	6	AC	ABC	6
B	BD	3	B	BD	3
B	BE	2	B	BE	2
B	BF	3	B	BF	3

BE	BG	3	BE	BG	3
BE	BH	3	BE	BH	3
BF	BI	3	BF	BI	3
BH	BJ	4	BH	ABC	4
BH	BK	3	BH	BK	3
C	CA	3	C	CA	3
C	CB	3	C	CB	3
C	CC	2	C	CC	2
CB	CD	3	CB	CD	3
CB	CE	2	CB	CE	2
CC	CF	3	CC	CF	3
CF	CG	3	CF	ABC	3
CF	CH	6	CF	CH	6

CONCLUSIONS

In this paper the authors have presented a model for examining the effect of commonality on optimal inventory levels. Beyond the specifics of the model, our analysis verifies the relationship between commonality and inventory levels in a multilevel system. The concept of inventory level is fundamental to single-product inventory analysis, but it is not obvious how to generalize that concept to the multiproduct situation. Our multiproduct model was only a three-product model, although our definitions of inventory level have obvious interpretations for the n-product case. However, the kind of commonality we examined is perhaps the simplest structural form. Analyzing the graph in the previous section we may conclude that-

- The introduction of commonality reduces the total inventory required
- The optimal stock of the common component is lower than the combined optimal stocks it replaces.
- The combined optimal stocks of product-specific components are higher with commonality than without

The outcomes are comply with the results of the studies of [12] and [13]. But their models investigates safety stock level for two end products with two product specific and one common component in a two-stage system under service level consideration.
For more than three end-items or more than three components per end-item or more than three levels, the possibilities for commonality are more numerous and much more complicated to analyze. Thus a direction for future research is a more complex product structure.

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